Root location of polynomials with totally nonnegative Hurwitz matrix

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For a given real polynomial

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \qquad a_0 > 0,$$

the $n \times n$ matrix $H_n(p) = (a_{2j-i})$ is called finite Hurwitz matrix, and the matrix $\mathcal{H}_\infty(p) = (a_{2j-i})_{i,j\in\mathbb{Z}}$ is the *infinite* Hurwitz matrix.

It is known [1, 2] that stability of the polynomial $p(z)$ (roots in the open left half-plane) implies the total nonnegativity of the matrices $\mathcal{H}_n(p)$ and $\mathcal{H}_\infty(p)$. However, the totally nonnegativity of the finite Hurwitz matrix $H_n(p)$ does not imply stability of $p(z)$.

In this talk, we show that the total nonnegativity of the matrix $\mathcal{H}_\infty(p)$ is equivalent to stability of the polynomial $p(z)$ (roots in the open left half-plane) and completely describe root location of the polynomial $p(z)$ whose finite Hurwitz matrix $H_n(p)$ is totally nonnegative.

This is a joint work with Mohammad Adm and Jürgen Garloff.

References

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[2] J.H.B. Kemperman, *A Hurwitz matrix is totally positive,* SIAM J. Math. Anal., **13**, 1982, pp. 331–341.

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