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Mathematical model of sea dynamics in a σ-coordinate system

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Abstract — The aim of this paper is to formulate a mathematical σ-model of thermohaline sea dynamics and its numerical solution. The novelty of the work is taking account of the nonhydrostatic effect, establishing the law of conservation for a complete nonlinear problem, and the generalization (for the nonhydrostatic case) of the numerical algorithm for solving the problem. The algorithm is based on the method of splitting with respect to physical processes. We describe nonhydrostatic effects at a separate splitting stage and introduce a new function describing the deviation of pressure from the hydrostatic one, which is calculated at an additional splitting stage. The elimination of this stage from the chain of split systems automatically leads to a special model case describing hydrostatic dynamics. Further, main attention is given to the barotropic dynamics problem. We formulate two finite difference algorithms of its solution: the first one by solving a linear hyperbolic system in terms of (u, v, ζ), the second algorithm by reducing it to the equation ζ.

The results of numerical modelling of sea and ocean dynamics suggest that the most important condition for increasing the adequacy of the models is improving their spatial resolution [3, 10]. An increase in the spatial resolution of the models requires enriching their physical content. Thus, for example, when the horizontal mesh sizes decrease, the condition of smallness of the vertical scale in described processes with respect to the horizontal one is no longer satisfied. In view of this, one has to abandon the hydrostatic approximation and consider a complete equation for the vertical velocity. Besides, lately the sea processes are often predicted by the series of numerical calculations in decreasing subdomains embedded in one another. This way of describing the processes in more detail brings the horizontal and vertical scales closer to each other. It requires the development of a hierarchical structure of models, which in the framework of a single algorithm allows one to use models of various levels of physical complexity.

The paper deals with the construction of a hierarchical sea dynamics model of this kind. The hierarchical structure of the model is based on the method of its numerical solution, viz. splitting it by physical processes [5, 6, 10]. It is the development of a model based on the general ocean circulation equations (primitive equations) written in the spherical σ-system of coordinates [9]. Since the base model is based on the splitting method, the module principle is inherent in its program realization. A separate splitting stage is represented as an separate program module.

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The main improvement of the model, viz. taking into account the nonhydrostatic effect was made possible by adding an additional splitting stage to the base model. This was achieved by the equivalent transformation of the original equations. The transformation is connected with the introduction of a new function describing the pressure change. This approach is analogous to the expansion of pressure into the sum of hydrostatic and nonhydrostatic components in [4, 8].

In the paper, we formulate the nonhydrostatic sigma model and the procedure for its numerical solution. The model is applied to simulate the baroclinic sea dynamics of the Sea of Okhotsk.

1. MODEL EQUATIONS, BOUNDARY AND INITIAL CONDITIONS

In the spherical coordinate system (λ, ϕ, z) the sea baroclinic dynamics equations have the form [8, 10]

$$\frac{\mathrm{d}u}{\mathrm{d}t} + mn\frac{\partial(m^{-1})}{\partial\phi}uv + nwu - \widehat{l}v + \widetilde{l}w = -\frac{m}{\rho_0}\frac{\partial p}{\partial\lambda} + \Lambda u \tag{1.1}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} - mn\frac{\partial(m^{-1})}{\partial\phi}uu + nwv + \widehat{l}u = -\frac{n}{\rho_0}\frac{\partial p}{\partial\phi} + \Lambda v \tag{1.2}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} - n(u^2 + v^2) - \widetilde{l}u = -\frac{1}{\rho_0}\left(\frac{\partial p}{\partial z} - g\rho\right) + \Lambda w \tag{1.3}$$

$$mn\left[\frac{\partial}{\partial\lambda}\left(\frac{u}{n}\right) + \frac{\partial}{\partial\phi}\left(\frac{v}{m}\right)\right] + \frac{\partial w}{\partial z} = 0 \tag{1.4}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \Lambda\rho. \tag{1.5}$$

Here

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + um\frac{\partial}{\partial\lambda} + vn\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z}$$

$$\Lambda * = mn\left(\frac{\partial}{\partial\lambda}\frac{m}{n}\mu_*\frac{\partial *}{\partial\lambda} + \frac{\partial}{\partial\phi}\frac{n}{m}\mu_*\frac{\partial *}{\partial\phi}\right) + \frac{\partial}{\partial z}\nu_*\frac{\partial *}{\partial z}$$

λ is the longitude, ϕ is the latitude, z is the vertical downward coordinate, u, v, w are the components of the velocity vector $\mathbf{u}$, p is the pressure, ρ is the potential density, ρ_0 is the given average density, ν_* is the vertical turbulent exchange coefficient, μ_* is the horizontal turbulent exchange coefficient, $\widehat{l} = 2\Omega\sin\phi$, $\widetilde{l} = 2\Omega\cos\phi$, Ω is the angular rotational velocity of the Earth, $m = 1/(R\cos\phi)$, $n = 1/R$, R is the Earth radius, g is the gravitational acceleration. Note that, for simplicity of presentation, instead of the temperature and salinity equations, we use a single equation for potential density.

Boundary conditions. The boundary conditions for (1.1)–(1.5) can be given as follows. Along the vertical for $z=\zeta(\lambda,\phi,t)$, we have

$$\nu_u\frac{\partial u}{\partial z}=-\frac{\tau_\lambda}{\rho_0},\qquad \nu_v\frac{\partial v}{\partial z}=-\frac{\tau_\phi}{\rho_0},\qquad \nu_\rho\frac{\partial \rho}{\partial z}=-Q_\rho^0 \qquad w=\frac{\mathrm{d}\zeta}{\mathrm{d}t},\qquad p=p_{\mathrm{atm}} \tag{1.6}$$

for $z=H(\lambda,\phi)$

$$\nu_u\frac{\partial u}{\partial z}=0,\qquad \nu_v\frac{\partial v}{\partial z}=0,\qquad \nu_\rho\frac{\partial \rho}{\partial z}=-Q_\rho^H,\qquad w=um\frac{\partial H}{\partial\lambda}+vn\frac{\partial H}{\partial\phi} \tag{1.7}$$

at the lateral surface Σ

$$\mu_\rho\frac{\partial\rho}{\partial\widehat{n}}=0,\qquad u=v=0. \tag{1.8}$$

Here ζ is the deviation of the sea level from the undisturbed surface, Q_ρ is the density flux, $\widehat{n}$ is the normal to Σ. Equations (1.1)–(1.5) are complemented by the initial conditions for $t=0$

$$\mathbf{u}=\mathbf{u}^0,\qquad \rho=\rho^0. \tag{1.9}$$

System (1.1)–(1.9) is considered in the time interval $(0,t]$ in the three-dimensional domain D.

2. POSING THE PROBLEM IN THE σ-COORDINATE SYSTEM

We rewrite equations (1.1)–(1.5) in the new coordinate system that differs from the spherical system by its vertical coordinate. We introduce the new vertical coordinate σ

$$\sigma=\frac{z-\zeta(\lambda,\phi,t)}{H(\lambda,\phi)-\zeta(\lambda,\phi,t)}.$$

A peculiarity of the coordinate σ-system is that it varies with time following the variations of the sea level and the bottom relief. Its upper and lower bounds are described by the simple equations $\sigma=0$, $\sigma=1$. If we assume that $(H(\lambda,\phi)-\zeta(\lambda,\phi,t))\geqslant \text{const}>0$, the new coordinate system is nondegenerate. It allows one to carry out calculations for sea basins comprising deep and shallow water areas by using the simple numerical representation of the model arrays along the vertical.

We rewrite (1.1)–(1.5) in the new system $(t_1,\lambda_1,\phi_1,\sigma)$: $t=t_1$, $\lambda=\lambda_1$, $\phi=\phi_1$, $\sigma=(z-\zeta)/(H-\zeta)$, taking into account that the relations hold

$$\frac{\partial}{\partial\lambda}=\frac{\partial}{\partial\lambda_1}-\frac{1}{h}\left[\frac{\partial\zeta}{\partial\lambda_1}+\sigma\frac{\partial}{\partial\lambda_1}h\right]\frac{\partial}{\partial\sigma}\equiv\frac{\partial}{\partial\lambda_1}-\frac{1}{h}\frac{\partial Z}{\partial\lambda_1}\frac{\partial}{\partial\sigma} \tag{2.1}$$

$$\frac{\partial}{\partial \phi} = \frac{\partial}{\partial \phi_1} - \frac{1}{h}\left[\frac{\partial \zeta}{\partial \phi_1} + \sigma \frac{\partial}{\partial \phi_1} h\right]\frac{\partial}{\partial \sigma} \equiv \frac{\partial}{\partial \phi_1} - \frac{1}{h}\frac{\partial Z}{\partial \phi_1}\frac{\partial}{\partial \sigma} \tag{2.2}$$

$$\frac{\partial}{\partial z} = \frac{1}{h}\frac{\partial}{\partial \sigma} \tag{2.3}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} - \frac{1}{h}\left[\frac{\partial \zeta}{\partial t_1} + \sigma \frac{\partial}{\partial t_1} h\right]\frac{\partial}{\partial \sigma} \equiv \frac{\partial}{\partial t_1} - \frac{1}{h}\frac{\partial Z}{\partial t_1}\frac{\partial}{\partial \sigma} \tag{2.4}$$

where $Z = \sigma h + \zeta$, $h = H - \zeta$.

Equations (1.1)–(1.5) are rewritten as

$$D_t u + h\left[mn\frac{\partial (m^{-1})}{\partial \phi} uv + nwu - \widehat{l}v + \widetilde{l}w\right] = -\frac{m}{\rho_0}\left[h\frac{\partial p}{\partial \lambda} - \frac{\partial Z}{\partial \lambda}\frac{\partial p}{\partial \sigma}\right] + \Lambda_1 u \tag{2.5}$$

$$D_t v - h\left[mn\frac{\partial (m^{-1})}{\partial \phi} uu - nwv - \widehat{l}u\right] = -\frac{n}{\rho_0}\left[h\frac{\partial p}{\partial \phi} - \frac{\partial Z}{\partial \phi}\frac{\partial p}{\partial \sigma}\right] + \Lambda_1 v \tag{2.6}$$

$$D_t w - h\left[n(u^2 + v^2) + \widetilde{l}u\right] = -\frac{1}{\rho_0}\left[\frac{\partial p}{\partial \sigma} - g\frac{\partial Z}{\partial \sigma}\rho\right] + \Lambda_1 w \tag{2.7}$$

$$mn\left[\frac{\partial}{\partial \lambda}\left(\frac{uh}{n}\right) + \frac{\partial}{\partial \phi}\left(\frac{vh}{m}\right)\right] + \frac{\partial \omega}{\partial \sigma} = \frac{\partial \zeta}{\partial t} \tag{2.8}$$

$$D_t \rho = \Lambda_1 \rho \tag{2.9}$$

where

$$D_t = h\frac{\partial}{\partial t} + muh\frac{\partial}{\partial \lambda} + nvh\frac{\partial}{\partial \phi} + \omega\frac{\partial}{\partial \sigma} \tag{2.10}$$

$$\omega = w - \left(m\frac{\partial Z}{\partial \lambda}u + n\frac{\partial Z}{\partial \phi}v + \frac{\partial Z}{\partial t}\right). \tag{2.11}$$

The turbulent exchange operator Λ_1 is written as

$$\begin{aligned}\Lambda_1 = {} & mn\left[\frac{\partial}{\partial \lambda}\left(\frac{m}{n}\mu h\frac{\partial}{\partial \lambda}\right) - \frac{\partial}{\partial \lambda}\left(\frac{m}{n}\mu\frac{\partial Z}{\partial \lambda}\frac{\partial}{\partial \sigma}\right) - \frac{\partial Z}{\partial \lambda}\frac{\partial}{\partial \sigma}\left(\frac{m}{n}\mu\frac{\partial}{\partial \lambda}\right)\right] \\ & + mn\left[\frac{\partial}{\partial \phi}\left(\frac{n}{m}\mu h\frac{\partial}{\partial \phi}\right) - \frac{\partial}{\partial \phi}\left(\frac{n}{m}\mu\frac{\partial Z}{\partial \phi}\frac{\partial}{\partial \sigma}\right) - \frac{\partial Z}{\partial \phi}\frac{\partial}{\partial \sigma}\left(\frac{n}{m}\mu\frac{\partial}{\partial \phi}\right)\right] \\ & + \frac{1}{h}\frac{\partial}{\partial \sigma}\left[\nu + \mu\left(m\frac{\partial Z}{\partial \lambda}\right)^2 + \mu\left(n\frac{\partial Z}{\partial \phi}\right)^2\right]\frac{\partial}{\partial \sigma}.\end{aligned} \tag{2.12}$$

The boundary conditions for the new vertical velocity ω on the sea surface and at the bottom have the simple form

$$\omega = 0 \qquad \text{for} \quad \sigma = 0, \quad \sigma = 1. \tag{2.13}$$

3. TOTAL ENERGY CONSERVATION LAW

Suppose in equations (2.5)–(2.9) there are no terms describing the turbulent exchange: $\Lambda_1 = 0$. In this case, we can use only one kinematic boundary condition for the velocity vector

$$(\mathbf{u}, \hat{n}) = 0. \tag{3.1}$$

We have

$$D_t u + Z_\sigma \left[mn \frac{\partial (m^{-1})}{\partial \phi} uv + nwu - \widehat{l}v + \widetilde{l}w \right] = -\frac{m}{\rho_0} \left[\frac{\partial Z}{\partial \sigma}\frac{\partial p}{\partial \lambda} - \frac{\partial Z}{\partial \lambda}\frac{\partial p}{\partial \sigma} \right] \tag{3.2}$$

$$D_t v - Z_\sigma \left[mn \frac{\partial (m^{-1})}{\partial \phi} uu - nwv - \widehat{l}u \right] = -\frac{n}{\rho_0} \left[\frac{\partial Z}{\partial \sigma}\frac{\partial p}{\partial \phi} - \frac{\partial Z}{\partial \phi}\frac{\partial p}{\partial \sigma} \right] \tag{3.3}$$

$$D_t w - Z_\sigma \left[n(u^2+v^2) + \widetilde{l}u \right] = -\frac{1}{\rho_0} \left[\frac{\partial p}{\partial \sigma} - g\frac{\partial Z}{\partial \sigma}\rho \right] \tag{3.4}$$

$$mn\left[\frac{\partial}{\partial \lambda}\left(\frac{uZ_\sigma}{n}\right) + \frac{\partial}{\partial \phi}\left(\frac{vZ_\sigma}{m}\right) \right] + \frac{\partial w}{\partial \sigma} - \frac{\partial}{\partial \sigma}\left(m\frac{\partial Z}{\partial \lambda}u + n\frac{\partial Z}{\partial \phi}v + \frac{\partial Z}{\partial t} \right) = \frac{\partial \zeta}{\partial t} \tag{3.5}$$

$$\frac{\partial Z_\sigma \rho}{\partial t} + \frac{\partial}{\partial \lambda}(mZ_\sigma u\rho) + \frac{\partial}{\partial \phi}(nZ_\sigma v\rho) + \frac{\partial}{\partial \sigma}\left[\left(w - m\frac{\partial Z}{\partial \lambda}u - n\frac{\partial Z}{\partial \phi}v - \frac{\partial Z}{\partial t} \right)\rho \right] = 0. \tag{3.6}$$

If we now take the inner product of (3.2)–(3.6) by the vector $(\rho_0 u, \rho_0 v, \rho_0 w, p, -gZ)$, with account taken of boundary conditions (3.1), we can show that the integral relation holds

$$\frac{\partial}{\partial t}\int_D \frac{\rho_0}{2} Z_\sigma (u^2+v^2+w^2)\mathrm{d}D - \frac{\partial}{\partial t}\int_D gZ_\sigma \rho Z \mathrm{d}D = \int_\Omega \frac{\partial \zeta}{\partial t} p_a \mathrm{d}\Omega, \quad \Omega = D \cap \{\sigma = 0\}. \tag{3.7}$$

If the atmospheric pressure does not vary with time, this relation expresses the total energy conservation law.

4. SPLITTING METHOD WITH RESPECT TO PHYSICAL PROCESSES. SEPARATION OF THE STAGES OF MOMENTUM TRANSFER AND NONHYDROSTATIC DYNAMICS

Equations (2.5)–(2.9) are nonlinear and have a complex structure. They describe the dynamics of several processes that greatly differ from one another by the extent of their time scale variability. The available algorithms of numerical solving of nonhydrostatic sea dynamics problems are complex and require high computational costs [4, 8]. As a rule, they use explicit schemes with very small time steps of the order of seconds and fractions of a second [4]. To increase the numerical efficiency, all nonhydrostatic models to a certain extent separate fast and slow processes. In the treatment of slow processes, one is trying to use the largest possible time steps. The typical drawback of many studies is that there is no distinct procedure for the separation of processes in time and matching solutions obtained at various calculation stages. Some isolated problems such as the problem of calculating the sea level and the nonhydrostatic pressure component are solved under artificial boundary conditions on a coastal boundary, which are not consistent with the original kinematic conditions.

In the present study, the methodological basis for the construction (and the numerical realization) of the sea dynamics model is the splitting method [5, 6]. The processes of different physical characters with different space-time scales are split into separate calculation stages. The system of equations solved at a separate splitting stage is simpler than the initial system (2.5)–(2.9). The boundary conditions for each separate problem result from the initial setting. The solution calculated at a particular stage is further used for solving the next stage.

The solution algorithm of system (2.5)–(2.9) is as follows: let us identity two main splitting stages: the transfer-diffusion of momentum, with account taken of the metric terms, and the adjustment of velocity and density fields. At the first stage we have

$$D_t u + Z_\sigma \left[mn \frac{\partial (m^{-1})}{\partial \phi} uv + nwu + \widetilde{l} w \right] = \Lambda_1 u \tag{4.1}$$

$$D_t v - Z_\sigma \left[mn \frac{\partial (m^{-1})}{\partial \phi} uu - nwv \right] = \Lambda_1 v \tag{4.2}$$

$$D_t w - Z_\sigma \left[n(u^2 + v^2) + \widetilde{l} u \right] = \Lambda_1 w. \tag{4.3}$$

To solve three-dimensional equations (4.1)–(4.3) we can use additional splitting based on individual space coordinates λ, ϕ, and σ.

At the stage of the adjustment of velocity and density fields we have

$$Z_\sigma \left(\frac{\partial u}{\partial t} - \widehat{l} v \right) = -\frac{m}{\rho_0} \left[\frac{\partial Z}{\partial \sigma} \frac{\partial p}{\partial \lambda} - \frac{\partial Z}{\partial \lambda} \frac{\partial p}{\partial \sigma} \right] \tag{4.4}$$

$$Z_\sigma \left(\frac{\partial v}{\partial t} + \widehat{l} u \right) = -\frac{n}{\rho_0} \left[\frac{\partial Z}{\partial \sigma} \frac{\partial p}{\partial \phi} - \frac{\partial Z}{\partial \phi} \frac{\partial p}{\partial \sigma} \right] \tag{4.5}$$

$$Z_\sigma \frac{\partial w}{\partial t} = -\frac{1}{\rho_0} \left[\frac{\partial p}{\partial \sigma} - g \frac{\partial Z}{\partial \sigma} \rho \right] \tag{4.6}$$

$$mn \left[\frac{\partial}{\partial \lambda} \left(\frac{Z_\sigma u}{n} \right) + \frac{\partial}{\partial \phi} \left(\frac{Z_\sigma v}{m} \right) \right] + \frac{\partial w}{\partial \sigma} - \frac{\partial}{\partial \sigma} \left(m \frac{\partial Z}{\partial \lambda} u + n \frac{\partial Z}{\partial \phi} v + \frac{\partial Z}{\partial t} \right) = \frac{\partial \zeta}{\partial t} \tag{4.7}$$

$$\frac{\partial Z_\sigma \rho}{\partial t} + \frac{\partial}{\partial \lambda}(mZ_\sigma u\rho) + \frac{\partial}{\partial \phi}(nZ_\sigma v\rho) + \frac{\partial}{\partial \sigma}\left[\left(w - m\frac{\partial Z}{\partial \lambda}u - n\frac{\partial Z}{\partial \phi}v - \frac{\partial Z}{\partial t}\right)\rho\right] = \Lambda_1 \rho. \tag{4.8}$$

Let us consider the method for solving equations (4.4)–(4.8). This system is simpler than the initial one but still remains rather complex. To solve it we can use the secondary splitting by isolating three problems that describe three physical processes. These are barotropic motion or sea level dynamics; the adjustment of velocity and density fields, and nonhydrostatic dynamics.

In order to separate the above problems we represent the pressure as

$$p = \widetilde{p} + g(H-\zeta)\rho_0\sigma + \int_0^\sigma gZ_\sigma \rho^1 \mathrm{d}\sigma, \qquad \rho^1 = \rho - \rho_0. \tag{4.9}$$

We introduce the new function $\widetilde{p}$ and two additional components. The first component describes pressure with a constant density ρ_0 and the second one the hydrostatic component of the pressure with residual density $(\rho - \rho_0)$.

The new function $\widetilde{p}$ is the deviation of normal pressure from its two main components mentioned above or, in other words, the nonhydrostatic part of the pressure. Taking into account (4.9), three first equations in system (4.4)–(4.8) are rewritten as

$$Z_\sigma\left(\frac{\partial u}{\partial t} - \widehat{l}v - mg\frac{\partial \zeta}{\partial \lambda}\right) = -\frac{m}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial}{\partial \lambda}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) - \frac{\partial Z}{\partial \lambda}\frac{\partial}{\partial \sigma}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) + \frac{\partial Z}{\partial \sigma}\frac{\partial \widetilde{p}}{\partial \lambda} - \frac{\partial Z}{\partial \lambda}\frac{\partial \widetilde{p}}{\partial \sigma}\right] \tag{4.10}$$

$$Z_\sigma\left(\frac{\partial v}{\partial t} + \widehat{l}v - ng\frac{\partial \zeta}{\partial \phi}\right) = -\frac{n}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial}{\partial \phi}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) - \frac{\partial Z}{\partial \phi}\frac{\partial}{\partial \sigma}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) + \frac{\partial Z}{\partial \sigma}\frac{\partial \widetilde{p}}{\partial \phi} - \frac{\partial Z}{\partial \phi}\frac{\partial \widetilde{p}}{\partial \sigma}\right] \tag{4.11}$$

$$Z_\sigma \frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \widetilde{p}}{\partial \sigma}. \tag{4.12}$$

Let us split the transformed system of equations (4.10)–(4.12), (4.7), (4.8). According to pressure expansion (4.9), taking (4.10)–(4.12), (4.7), and (4.8), we separate out three problems to be solved at separate stages. The first problem is the sea level dynamics problem

$$\frac{\partial U}{\partial t} - \widehat{l}V - mg\frac{\partial \zeta}{\partial \lambda} = 0 \tag{4.13}$$

$$\frac{\partial V}{\partial t} + \widehat{l}U - ng\frac{\partial \zeta}{\partial \phi} = 0 \tag{4.14}$$

$$\frac{\partial \zeta}{\partial t} = mn\left[\frac{\partial}{\partial \lambda}\left(\frac{Z_\sigma U}{n}\right) + \frac{\partial}{\partial \phi}\left(\frac{Z_\sigma V}{m}\right)\right] \tag{4.15}$$

where

$$U = \int_0^1 u\mathrm{d}\sigma, \qquad V = \int_0^1 v\mathrm{d}\sigma \tag{4.16}$$

the second one is the problem of the adjustment of velocity and density fields

$$Z_\sigma\left[\frac{\partial u}{\partial t} - \widehat{l}(v - V)\right] = -\frac{m}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial}{\partial \lambda}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) - \frac{\partial Z}{\partial \lambda}\frac{\partial}{\partial \sigma}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right)\right] \tag{4.17}$$

$$Z_\sigma\left[\frac{\partial v}{\partial t} + \widehat{l}(u - U)\right] = -\frac{n}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial}{\partial \phi}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right) - \frac{\partial Z}{\partial \phi}\frac{\partial}{\partial \sigma}\left(gZ_\sigma \int_0^\sigma \rho^1 \mathrm{d}\sigma\right)\right] \tag{4.18}$$

$$\frac{\partial w}{\partial t} = 0 \tag{4.19}$$

$$\frac{\partial Z_\sigma \rho^1}{\partial t} + \frac{\partial}{\partial \lambda}(mZ_\sigma u\rho^1) + \frac{\partial}{\partial \phi}(nZ_\sigma v\rho^1) + \frac{\partial \omega\rho^1}{\partial \sigma} = \Lambda_1 \rho^1. \tag{4.20}$$

Finally, the third problem is the nonhydrostatic dynamics problem

$$Z_\sigma \frac{\partial u}{\partial t} = -\frac{m}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial \widetilde{p}}{\partial \lambda} - \frac{\partial Z}{\partial \lambda}\frac{\partial \widetilde{p}}{\partial \sigma}\right] \tag{4.21}$$

$$Z_\sigma \frac{\partial v}{\partial t} = -\frac{n}{\rho_0}\left[\frac{\partial Z}{\partial \sigma}\frac{\partial \widetilde{p}}{\partial \phi} - \frac{\partial Z}{\partial \phi}\frac{\partial \widetilde{p}}{\partial \sigma}\right] \tag{4.22}$$

$$Z_\sigma \frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \widetilde{p}}{\partial \sigma} \tag{4.23}$$

$$mn\left[\frac{\partial}{\partial \lambda}\left(\frac{Z_\sigma(u - U)}{n}\right) + \frac{\partial}{\partial \phi}\left(\frac{Z_\sigma(v - V)}{m}\right)\right] + \frac{\partial w}{\partial \sigma} - \frac{\partial}{\partial \sigma}\left(m\frac{\partial Z}{\partial \lambda}u + n\frac{\partial Z}{\partial \phi}v + \frac{\partial Z}{\partial t}\right) = 0. \tag{4.24}$$

Thus, the full sea dynamics model consists of four split subsystems: (4.1)–(4.3), (4.13)–(4.15), (4.17)–(4.20), (4.21)–(4.24).

In many cases we can use simplified models rather than the above full sea dynamics model. The conventional approximations in oceanology are: (a) hydrostatic approximation resulting from the fact that the horizontal motion scale is much larger than vertical motion ($L \gg H$) and (b) long-wave approximation $H \gg \zeta$. Large-scale sea and ocean circulation models are generally formulated in the framework of these approximations [1–3, 6, 7, 9]. They are described by the so-called primitive equation system which results from (2.5)–(2.9). In (2.5)–(2.9), we take $Z = \sigma H$. The terms dependent on w are eliminated from the first two equations, the third equation reduces to the hydrostatic relation $\partial p/\partial\sigma = gH\rho$. Algorithmically, this case can be adjusted to our approach rather simply. The first three stages of splitting with respect to physical processes do not change, the only difference concerns the equation for w. At the final stage, we take $\widetilde{p} = 0$ and it reduces to the calculation of the new vertical velocity ω from the equation

$$mn\left[\frac{\partial}{\partial\lambda}\left(\frac{Z_\sigma(u-U)}{n}\right)+\frac{\partial}{\partial\phi}\left(\frac{Z_\sigma(v-V)}{m}\right)\right]+\frac{\partial\omega}{\partial\sigma}=0. \tag{4.25}$$

One of the most important practical problems of sea and ocean dynamics is a problem of calculating the sea level. It is the prediction of tidal waves in adjacent seas and in the World ocean, the calculation of wind flows and storm surges and so on. Particular interest in the problem has been shown in the past few years because of the possibility to measure the sea level by satellites. It became possible to use satellite altimetry for the verification of the numerical models, to pose and solve data assimilation problems. Let us consider the finite difference method for solving the sea level problem.

5. FORMULATION OF THE SEA LEVEL PROBLEM

The equations describing the evolution of the vertical-average components of the velocity U, V and the sea level ζ have the form

$$\frac{\partial U}{\partial t} - lV = mg\frac{\partial\zeta}{\partial\lambda} - RU \tag{5.1}$$

$$\frac{\partial V}{\partial t} + lU = ng\frac{\partial\zeta}{\partial\phi} - RV \tag{5.2}$$

$$\frac{1}{mn}\frac{\partial\zeta}{\partial t} - \left[\frac{\partial}{\partial\lambda}\left(\frac{HU}{n}\right)+\frac{\partial}{\partial\phi}\left(\frac{HV}{m}\right)\right]=0. \tag{5.3}$$

These equations follow from (4.13)–(4.15), with account taken of the approximation $H \gg \zeta$, and with additional terms RU, RV describing the bottom friction. Here R is the bottom friction coefficient. It is either a given positive value or a quadratic function of U, V.

The boundary conditions, in addition to equations (5.1)–(5.3) include

$$(\mathbf{U},\widehat{n}) = 0, \qquad \partial D_1 \quad (\mathbf{U},\widehat{n}) = \mathbf{f}, \qquad \partial D_2 \tag{5.4}$$

where $\mathbf{U} = (U,V)$, ∂D_1 is the coastal boundary and ∂D_2 is the liquid contour. Equations (5.1)–(5.3) are approximated implicitly on the time interval $t_j \leqslant t \leqslant t_{j+1}$. Further, to solve the problem obtained we can use two methods. The first method is to solve the derived system of algebraic equations in terms of (U,V,ζ). The second method is to eliminate the velocity components and reduce the problem to the equation for the sea level ζ, with boundary conditions resulting from (5.4). Let us consider both methods in more detail.

Method 1. Solution of the problem in terms of U,V,ζ. Suppose the domain of the solution to the problem consists of rectangles, the grid steps are nonuniform such that $hx = hx(\lambda)$, $hy = hy(\phi)$. We approximate the equations of motion implicitly with respect to time, multiply them by mn, taking into account that the element of the domain is $\mathrm{d}D = \frac{1}{mn}\mathrm{d}\lambda\,\mathrm{d}\phi$, and integrate the equations with respect to the corresponding cells. We have

$$\int_{D_{i+1/2,j}} mn\left[(\delta+R)U - lV - mg\frac{\partial\zeta}{\partial\lambda} - \delta U^0\right]\mathrm{d}D_{i+1/2,j} = 0 \tag{5.5}$$

$$\int_{D_{i,j+1/2}} mn\left[(\delta+R)V + lU - ng\frac{\partial\zeta}{\partial\phi} - \delta V^0\right]\mathrm{d}D_{i,j+1/2} = 0, \qquad \delta = \frac{1}{\tau}. \tag{5.6}$$

Suppose that the function U is piecewise constant and defined at the point $(i+1/2,j)$: $U_{i+1/2,j}$ and V at the point $(i,j+1/2)$: $V_{i,j+1/2}$. The function l is piecewise constant given on a finer grid at the points $(i+1/4, j+1/4)$. The coefficient n is given at the points of the definition of U, and m at the points V. Then (5.5), (5.6) yield

$$\begin{aligned}
& hx_{i+1/2}\frac{hy_{j+1/2}+hy_{j-1/2}}{2}((\delta+R)U)_{i+1/2,j} \\
& -l_{i+3/4,j+1/4}\frac{hx_{i+1/2}hy_{j+1/2}}{4}(LcvV)_{i+1,j+1/2} \\
& -l_{i+1/4,j-1/4}\frac{hx_{i+1/2}hy_{j-1/2}}{4}(LcvV)_{i,j-1/2} \\
& -l_{i+1/4,j+1/4}\frac{hx_{i+1/2}hy_{j+1/2}}{4}(LcvV)_{i,j+1/2} \\
& -l_{i+3/4,j-1/4}\frac{hx_{i+1/2}hy_{j-1/2}}{4}(LcvV)_{i+1,j-1/2} \\
& = \frac{hy_{j+1/2}+hy_{j-1/2}}{2}hx_{i+1/2}\left(m_{i+1/2,j}\,g\frac{\zeta_{i+1,j}-\zeta_{i,j}}{hx_{i+1/2}} + \delta U^0_{i+1/2,j}\right)
\end{aligned} \tag{5.7}$$

$$
\begin{aligned}
& hy_{j+1/2}\frac{hx_{i+1/2}+hx_{i-1/2}}{2}((\delta+R)V)_{i,j+1/2} \\
& +l_{i-1/4,j+1/4}\frac{hx_{i-1/2}hy_{j+1/2}}{4}(LcuU)_{i-1/2,j} \\
& +l_{i+1/4,j+3/4}\frac{hx_{i+1/2}hy_{j+1/2}}{4}(LcuU)_{i+1/2,j+1} \\
& +l_{i-1/4,j+3/4}\frac{hx_{i-1/2}hy_{j+1/2}}{4}(LcuU)_{i-1/2,j+1} \\
& +l_{i+1/4,j+1/4}\frac{hx_{i+1/2}hy_{j+1/2}}{4}(LcuU)_{i+1/2,j} \\
& =\frac{hx_{i+1/2}+hx_{i-1/2}}{2}hy_{j+1/2}\left(n_{i,j+1/2}g\frac{\zeta_{i,j+1}-\zeta_{i,j}}{hy_{j+1/2}}+\delta V^0_{i,j+1/2}\right). \qquad (5.8)
\end{aligned}
$$

The approximation of the continuity equation has the form

$$
\begin{aligned}
& \left(\frac{1}{mn}\right)_{i,j}\delta(\zeta_{i,j}-\zeta^0_{i,j})-\left[\left(\frac{HU}{n}Lcu\right)_{i+1/2,j}-\left(\frac{HU}{n}Lcu\right)_{i-1/2,j}\right]\frac{2}{hx_{i+1/2}+hx_{i-1/2}} \\
& -\left[\left(\frac{HV}{m}Lcv\right)_{i,j+1/2}-\left(\frac{HV}{m}Lcv\right)_{i,j-1/2}\right]\frac{2}{hy_{j+1/2}+hy_{j-1/2}}=0. \qquad (5.9)
\end{aligned}
$$

Here $Lcu_{i+1/2,j}$, $Lcv_{i,j+1/2}$ are mask arrays equal to unity at the points of the calculated domain and equal to zero beyond its bounds. The problem reduces to the solution of the system of algebraic equations (5.7)–(5.9).

Method 2. Reducing the problem to the equation for the sea level. As before, equations (5.1)–(5.3) are approximated implicitly on the time interval $t_j \leqslant t \leqslant t_{j+1}$. However, the problem further reduces to the equation for the sea level ζ, with boundary conditions following from (5.4). The solution algorithm of the problem is to calculate the velocity components U, V and solve the elliptic (at each time step) equation for the sea level. From (5.1)–(5.3) we have

$$
(\delta+R)U-lV\cdot Lcv=mg\frac{\partial\zeta}{\partial\lambda}+\delta U^0 \qquad (5.10)
$$

$$
(\delta+R)V+lU\cdot Lcu=ng\frac{\partial\zeta}{\partial\phi}+\delta V^0 \qquad (5.11)
$$

$$
\frac{1}{mn}\delta(\zeta-\zeta^0)-\left[\frac{\partial}{\partial\lambda}\left(\frac{HU}{n}\cdot Lcu\right)+\frac{\partial}{\partial\phi}\left(\frac{HV}{m}\cdot Lcv\right)\right]=0. \qquad (5.12)
$$

We rewrite these equations in the form resolved with respect to U, V:

$$
\Delta U=(\delta+R)\left(mg\frac{\partial\zeta}{\partial\lambda}+\delta U^0\right)+l\left(ng\frac{\partial\zeta}{\partial\phi}+\delta V^0\right)\cdot Lcv \qquad (5.13)
$$

$$\Delta V = (\delta + R)\left(ng\frac{\partial\zeta}{\partial\phi} + \delta V^0\right) - l\left(mg\frac{\partial\zeta}{\partial\phi} + \delta U^0\right)\cdot Lcu \tag{5.14}$$

$$\Delta = (\delta + R)^2 + l^2 \cdot Lcu \cdot Lcv.$$

Hence,

$$U = \frac{(\delta + R)}{\Delta}\left(mg\frac{\partial\zeta}{\partial\lambda} + \delta U^0\right) + \frac{l}{\Delta}\left(ng\frac{\partial\zeta}{\partial\phi} + \delta V^0\right)\cdot Lcv \tag{5.15}$$

$$V = \frac{(\delta + R)}{\Delta}\left(ng\frac{\partial\zeta}{\partial\phi} + \delta V^0\right) - \frac{l}{\Delta}\left(mg\frac{\partial\zeta}{\partial\lambda} + \delta U^0\right)\cdot Lcu. \tag{5.16}$$

To calculate the sea level, we write the expressions for the functions $HU/n, HV/m$

$$\frac{HU}{n} = \frac{m}{n}H^{(R)}\frac{\partial\zeta}{\partial\lambda} + H^{(l)}\frac{\partial\zeta}{\partial\phi}\cdot Lcv + \frac{\delta}{gn}\left(H^{(R)}U^0 + H^{(l)}V^0\cdot Lcv\right) \tag{5.17}$$

$$\frac{HV}{m} = \frac{n}{m}H^{(R)}\frac{\partial\zeta}{\partial\phi} - H^{(l)}\frac{\partial\zeta}{\partial\lambda}\cdot Lcu + \frac{\delta}{gm}\left(H^{(R)}V^0 - H^{(l)}U^0\cdot Lcu\right) \tag{5.18}$$

where

$$H^{(R)} = \frac{gH(\delta + R)}{\Delta}, \quad H^{(l)} = \frac{gHl}{\Delta}. \tag{5.19}$$

We approximate (5.17), (5.18) on a nonuniform grid with steps $hx_{i+1/2}$, $hy_{j+1/2}$ to obtain

$$hx_{i+1/2}\frac{hy_{j+1/2} + hy_{j-1/2}}{2}\left(\frac{HU}{n}\right)_{i+1/2,j}$$

$$= hx_{i+1/2}\frac{hy_{j+1/2} + hy_{j-1/2}}{2}H^{(R)}_{i+1/2,j}\left[\left(\frac{m}{n}\right)_{i+1/2,j}\frac{\zeta_{i+1,j} - \zeta_{i,j}}{hx_{i+1/2}} + \frac{\delta}{g}\left(\frac{U^0}{n}\right)_{i+1/2,j}\right]$$

$$+ H^{(l)}_{i+1/2,j+1/2}\frac{hx_{i+1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i+1,j+1} - \zeta_{i+1,j}}{hy_{j+1/2}} + \frac{\delta}{g}\left(\frac{V^0}{n}\right)_{i+1,j+1/2}\right]Lcv_{i+1,j+1/2}$$

$$+ H^{(l)}_{i+1/2,j-1/2}\frac{hx_{i+1/2}hy_{j-1/2}}{4}\left[\frac{\zeta_{i,j} - \zeta_{i,j-1}}{hy_{j-1/2}} + \frac{\delta}{g}\left(\frac{V^0}{n}\right)_{i,j-1/2}\right]Lcv_{i,j-1/2} \tag{5.20}$$

$$+H^{(l)}_{i+1/2,j+1/2}\frac{hx_{i+1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i,j+1}-\zeta_{i,j}}{hy_{j+1/2}}+\frac{\delta}{g}\left(\frac{V^0}{n}\right)_{i,j+1/2}\right]Lcv_{i,j+1/2}$$

$$+H^{(l)}_{i+1/2,j-1/2}\frac{hx_{i+1/2}hy_{j-1/2}}{4}\left[\frac{\zeta_{i+1,j}-\zeta_{i+1,j-1}}{hy_{j-1/2}}+\frac{\delta}{g}\left(\frac{V^0}{n}\right)_{i+1,j-1/2}\right]Lcv_{i+1,j-1/2};$$

$$hy_{j+1/2}\frac{hx_{i+1/2}+hx_{i-1/2}}{2}\left(\frac{HV}{m}\right)_{i,j+1/2}$$

$$=hy_{j+1/2}\frac{hx_{i+1/2}+hx_{i-1/2}}{2}H^{(R)}_{i,j+1/2}\left[\left(\frac{n}{m}\right)_{i,j+1/2}\frac{\zeta_{i,j+1}-\zeta_{i,j}}{hy_{j+1/2}}+\frac{\delta}{g}\left(\frac{V^0}{m}\right)_{i,j+1/2}\right]$$

$$-H^{(l)}_{i-1/2,j+1/2}\frac{hx_{i-1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i,j}-\zeta_{i-1,j}}{hx_{i-1/2}}+\frac{\delta}{g}\left(\frac{U^0}{m}\right)_{i-1/2,j}\right]Lcu_{i-1/2,j}$$

$$-H^{(l)}_{i+1/2,j+1/2}\frac{hx_{i+1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i+1,j+1}-\zeta_{i,j+1}}{hx_{i+1/2}}+\frac{\delta}{g}\left(\frac{U^0}{m}\right)_{i+1/2,j+1}\right]Lcu_{i+1/2,j+1}$$

$$-H^{(l)}_{i-1/2,j+1/2}\frac{hx_{i-1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i,j+1}-\zeta_{i-1,j+1}}{hx_{i-1/2}}+\frac{\delta}{g}\left(\frac{U^0}{m}\right)_{i-1/2,j+1}\right]Lcu_{i-1/2,j+1}$$

$$-H^{(l)}_{i+1/2,j+1/2}\frac{hx_{i+1/2}hy_{j+1/2}}{4}\left[\frac{\zeta_{i+1,j}-\zeta_{i,j}}{hx_{i+1/2}}-\frac{\delta}{g}\left(\frac{U^0}{m}\right)_{i+1/2,j}\right]Lcu_{i+1/2,j}. \quad (5.21)$$

The approximation of the continuity equation is

$$\left(\frac{1}{mn}\right)_{i,j}\delta(\zeta_{i,j}-\zeta^0_{i,j})-\left[\left(\frac{HU}{n}Lcu\right)_{i+1/2,j}-\left(\frac{HU}{n}Lcu\right)_{i-1/2,j}\right]\frac{2}{hx_{i+1/2}+hx_{i-1/2}}$$

$$-\left[\left(\frac{HV}{m}Lcv\right)_{i,j+1/2}-\left(\frac{HV}{m}Lcv\right)_{i,j-1/2}\right]\frac{2}{hy_{j+1/2}+hy_{j-1/2}}=0. \quad (5.22)$$

Substituting (5.20), (5.21) in (5.22), we obtain the nine-point equation for the sea level

$$r1\cdot\zeta_{i,j-1}+r0\cdot\zeta_{i,j}+r2\cdot\zeta_{i,j+1}+r3\cdot\zeta_{i-1,j}+r4\cdot\zeta_{i+1,j}$$
$$+r5\cdot\zeta_{i-1,j-1}+r6\cdot\zeta_{i+1,j-1}+r7\cdot\zeta_{i+1,j+1}+r8\cdot\zeta_{i-1,j+1}=F_{i,j} \quad (5.23)$$

with the corresponding expressions for the coefficients and the right-hand side following from (5.20)–(5.22).

At each time step, scheme (5.23) approximates the equation for the sea level function

$$\frac{\delta}{mn}\zeta-\frac{\partial}{\partial\lambda}\frac{mH^{(R)}}{n}\frac{\partial\zeta}{\partial\lambda}-\frac{\partial}{\partial\phi}\frac{nH^{(R)}}{m}\frac{\partial\zeta}{\partial\phi}-\frac{\partial}{\partial\lambda}H^{(l)}\frac{\partial\zeta}{\partial\phi}+\frac{\partial}{\partial\phi}H^{(l)}\frac{\partial\zeta}{\partial\lambda}=F \quad (5.24)$$

$$F=\frac{\delta}{mn}\zeta^0+\frac{\delta}{g}\left(\frac{\partial}{\partial\lambda}\frac{H^{(R)}}{n}U^0+\frac{\partial}{\partial\lambda}\frac{H^{(l)}}{n}V^0+\frac{\partial}{\partial\phi}\frac{H^{(R)}}{m}V^0-\frac{\partial}{\partial\phi}\frac{H^{(l)}}{m}U^0\right). \quad (5.25)$$

The boundary condition for equation (5.24) follows from (5.4) and is natural in the variational sense. Its form follows from expressions (5.15), (5.16) for U, V.

6. NUMERICAL MODELLING OF THE CIRCULATION IN THE SEA OF OKHOTSK

The Sea of Okhotsk is a semi-enclosed sea basin located in mid- latitudes in the north-west part of the Pacific Ocean. It joins the Sea of Japan in the south-west, it is bounded in the south by the Kuril Islands and in the east by the Kamchatka Peninsula. The Sakhalin Island bounds the sea basin in the west. The north boundary of the sea is the continental coast. The circulation in the Sea of Okhotsk is formed under the effect of the monsoon wind, the tidal wave incoming through the straits of the Kuril Islands, and the thermohaline factors.

The main geographical features of the Sea of Okhotsk are:

- Large depth gradients (the basin comprises shallow bays and deep areas, the sea depth varies from several meters to 2–3 km).
- The presence of a wide north-west shelf about 220 m in depth.
- Strong changeable winds.
- A large number of straits and strong currents.
- High tides (the wave height in the Penzhinskaya Bay reaches 13 m).

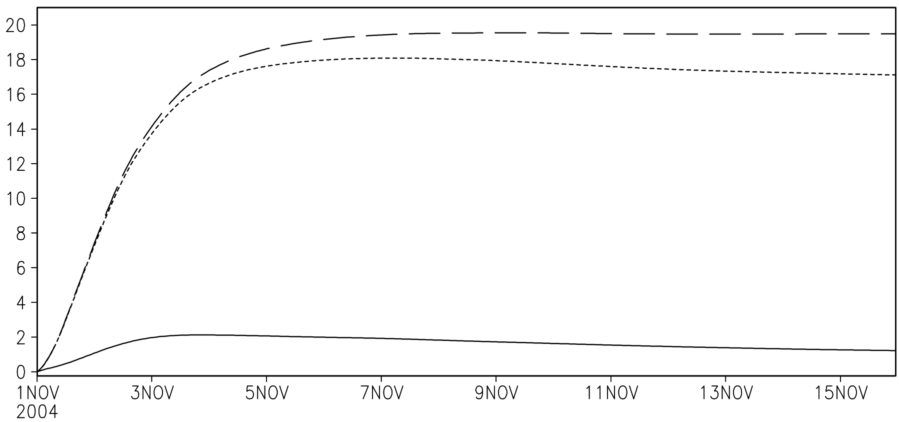

Figure 1. Time series of the barotropic kinetic energy, averaged over the whole domain. November: — thermohaline circulation, ··· wind circulation, - - - wind and thermohaline circulation.

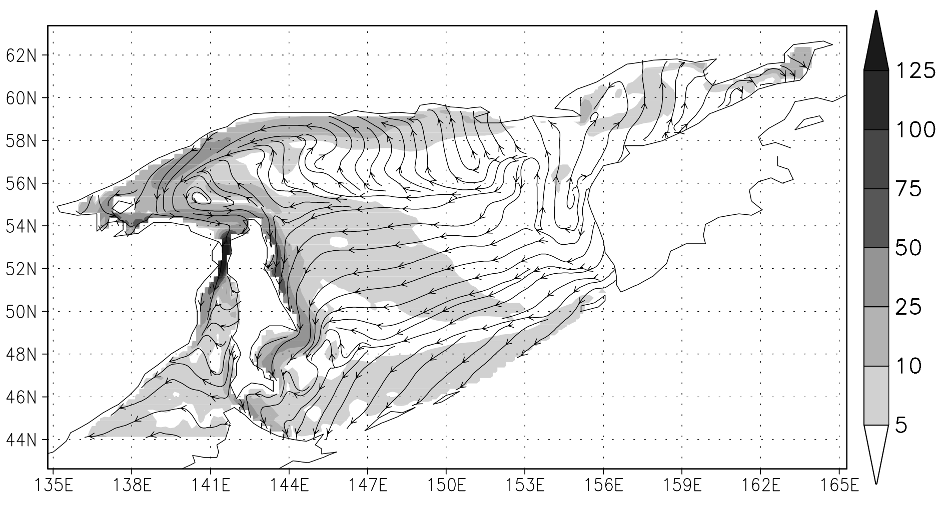

Figure 2. Wind currents at 6-m depth (November).

All the above factors cause significant difficulties in the modelling and prediction of currents in the Sea of Okhotsk. We should emphasize that, besides scientific interest, the simulation of the processes in the Sea of Okhotsk is of interest for such applied sectors as fishery, prospecting, oil production, navigation, and use of marine structures.

One of the most interesting problems of examining the dynamics of the Sea of Okhotsk is estimating the contribution of various factors to the formation of the dynamics of surface flows and the sea level. The numerical experiment using the

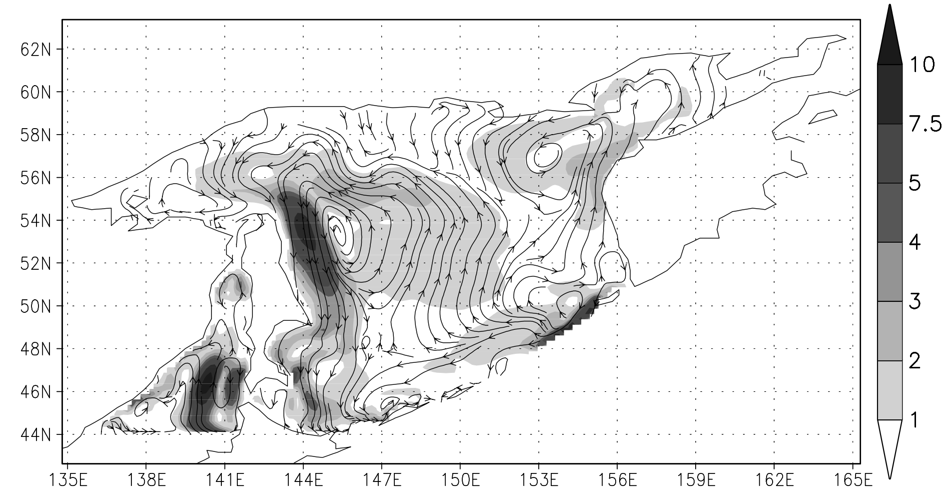

Figure 3. Thermohaline currents at 6-m depth (November).

above numerical model is designed exactly for this problem. The calculations were carried out with account taken of the conventional approximations of hydrostatics and long waves ($L \gg H, H \gg \zeta$).

The climatic monthly average wind fields were used as atmospheric forcing. The effects of the tide and the thermohaline factors were taken into account in the model. The lunar diurnal wave K_1 that is most pronounced in the aquatorium was considered as a tidal wave. The wave was generated on the open south-east boundary (in the Kuril straits). The wave amplitude was taken to be 50 cm. In the general case, we calculated all the hydrologic fields of velocity, density (temperature and salinity) and the sea level. Several numerical experiments for different months of the year were carried out. The calculations differed by taking account of different factors.

The first series of calculations was carried out under the winter monsoon conditions (mid-November). Only the wind effect was taken into account in the first run. The density was taken to be constant, the tide was absent. Only the thermohaline factors (the wind and the tide were absent) were taken into account in the second run and only the tidal wave K_1 in the third run. The joint effect of the wind and the thermohaline factors was estimated in the fourth run. The same series of calculations was carried out for the summer monsoon conditions (mid-June). The computational results suggest the following.

(1) The experiments show that the tidal flows for the diurnal wave K_1 have the highest energy. The average barotropic kinetic energy for the wave K_1 is about 55 $(cm/s)^2$. These values for the wind and thermohaline circulation are, respectively, 20 and 2 $(cm/s)^2$ in November (Fig. 1), 1 and 3.4 $(cm/s)^2$ in June. Instantaneous tidal flows reach 390 cm/s in the Penzhinskaya Bay. The maximum amplitude of the tidal wave is attained there too and is about 4.5 m.

(2) The residual (averaged over the tidal period) tidal flows are not large. Their

mean value is about 7 cm/s near the southern coast of Sakhalin, 3 cm/s in the open sea, and the maximum value (in the Penzhinskaya Bay) reaches 33 cm/s.

(3) The residual flows are primarily formed by the wind. The maximum velocity of the wind flows in the upper sea layer is observed near the coast of Sakhalin. In November the velocity is higher than 120 cm/s at a depth of 6 m (Fig. 2) and about 20 cm/s at a depth of 100 m. The amplitude of the sea level fluctuations is up to 70 cm in November and about 8 cm in June.

(4) The contribution of the thermohaline factor to the flow formation in the upper sea layer is not large. In the absence of the wind and the tide the velocities of thermohaline flows at a depth of 6 m are about 13 cm/s in November (Fig. 3) and 12 cm/s in June and about 10–12 cm/s at a depth of 100 m. The characteristic amplitude of the sea level fluctuations is not large, i.e., about 7 cm.

REFERENCES

1. K. Bryan, A numerical method for the study of the circulation of the world ocean. *J. Comp. Phys.* (1969) **4**, 347–376.
2. S. M. Griffies, C. Boening, F. O. Bryan, E. P. Chassignet, R. Gerdes, H. Hasumi, A. Hirst, A.-M. Treguier, and D. Webb, Developments in ocean climate modelling. *Ocean Modelling* (2000) **2**, 123–192.
3. D. B. Haidvogel and A. Beckmann, *Numerical Ocean Circulation Modelling*. Imperial Colledge Press, London, 1999.
4. Y. Kanarska and V. Maderich, A nonhydrostatic numerical model for calculating free-surface stratified flows. *Ocean Dynamics* (2003) **53**, 176–185.
5. G. I. Marchuk, *Methods of Computational Mathematics*. Nauka, Moscow, 1980.
6. G. I. Marchuk and A. S. Sarkisyan, *Mathematical Modelling of Ocean Circulation*. Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo, 1988.
7. G. I. Marchuk, J. Schroeter, and V. B. Zalesny, Numerical study of the global ocean equilibrium circulation. *Russ. J. Numer. Anal. Math. Modelling* (2003) **18**, No. 4, 307–335.
8. J. Marshall, C. Hill, L. Perelman, and A. Adcroft, Hydrostatic, quasi-hydrostatic, and nonhydrostatic ocean modelling. *J. Geophys. Res.* (1997) **102**, No. C3, 5733–5752.
9. V. B. Zalesny, Numerical simulation and analysis of the sensitivity of large-scale ocean dynamics. *Russ. J. Numer. Anal. Math. Modelling* (1996) **11**, No. 6, 421–443.
10. V. B. Zalesny, R. Tamsalu, and T. Kullac, Nonhydrostatic model of the marine circulation. *Oceanology* (2004) **44**, No. 4, 495–506.