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MARINE PHYSICS

On the Self-Adjusted Description of the Atmospheric Boundary Layer, Wind Waves, and Sea Currents

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Abstract—The problems of the self-adjusted description of the atmospheric boundary layer, wind waves, and sea currents are considered. The atmospheric and sea dynamics are calculated independently with models of different complexity. The models are coupled with the model of wind waves. The model of wind waves is based on a strictly directional approximation and a special procedure for selecting the wave perturbations in the atmospheric and sea boundary layers. The model was used for the calculation of the characteristics of the fluxes that appear in the boundary conditions at the sea surface. Two new results were obtained: the theoretical estimate of the contribution of the wind waves to the flux of the turbulent kinetic energy at the sea surface and the model estimate of the effect of wind waves on the marine dynamics. The latter was obtained from a numerical experiment with the use of a three-dimensional nonhydrostatic model of the thermohaline dynamics of the sea. The numerical experiment is carried out for a rectangular domain that simulates the middle part of the Baltic Sea. The numerical calculations show a pronounced contribution of the effect of wind waves to the dynamics of the upper sea layer.

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1. INTRODUCTION

In the present study, we discuss the problems of coupling the theoretical models describing the principal dynamical effects in the vicinity of the sea–air interface: wind, waves, and sea currents. The present-day models of the atmospheric boundary layer and the upper sea layer (see [11, 12, 16, 17, 20, 23, 26]) are rather general: they use the equation of the balance of turbulent energy [14] for closing the Reynolds equations and, hence, need the momentum and energy fluxes through the sea–air interface to be given. The equations for the fluxes are formulated on the basis of certain physical concepts and include a set of parameters. Important parameters are the current velocity U_* and the roughness of the water surface z_0, which in turn depend on the wind waves. In this connection, the following should be noted. First, all the relationships between z_0 and the parameters of the wind waves used are rather hypothetical, which makes the estimates of the wind stress τ_a unreliable in the boundary condition for the momentum flux at the sea surface. When one describes sea currents, the effect of wind waves is either ignored (the total momentum flux τ_a from the atmosphere is assumed to be transformed into a drift current, although, actually, a part of it should be transformed into wind waves) or taken into account empirically with no reliable theoretical base. Second, when one imposes the boundary conditions at the sea surface for the flux of the turbulent energy, the effect of wind waves is also assessed rather arbitrarily [16].

In the present study, we propose a general method for coupling the units for modeling the atmospheric circulation and dynamics of the ocean at the sea–air interface with the use of an additional wave unit. The model units describing the atmospheric and sea dynamics are calculated independently and can have different levels of complexity. The interaction between the units is implemented with the use of a special wave unit. The unit of wave dynamics is based on the transport equation for the spectrum of wind waves in a narrow directional approximation and a special procedure for selecting the wave perturbations in the boundary layers in the air and sea. In addition to the determination of the spectrum of the wind waves, it is used for coupling the models of turbulent boundary layers in the air and sea and for calculating the characteristics of the fluxes that are necessary for imposing the boundary conditions at the interface. This results in a closed self-adjusted description of wind, waves, and sea currents, which takes into account the adjustment of all these processes.

In recent years, more and more attention is being paid to the construction of models that take into account the interaction between waves and sea currents. The effect of surface wave breaking on the turbulent dynamics of the upper layer [16, 20] is being studied, and coupled models for waves and currents have begun to be developed [23]. The present study is devoted to the development of this line of research. Two new results should be highlighted. First, we obtained a theoretical estimation of the contribution of wind waves to the flux of the turbulent kinetic energy at the sea surface. Second, we obtained a model estimation of the effect of wind waves on the dynamics of the sea. The estimation is made on the basis of a numerical experiment with the use of a three-dimensional nonhydrostatic numerical model of the thermohaline dynamics of the sea. The calculations show a pronounced contribution of wind waves to the dynamics of the upper sea layer.

2. ATMOSPHERIC UNIT

For the atmospheric unit, we can use an updated model for the atmospheric dynamics or a simpler version. As an example, let us consider the model of the surface atmospheric layer (SAL) proposed in [5]. According to this model, the current velocity U_* and the roughness parameter z_0 are represented in terms of the wind velocity U_g and the spatial spectrum of wind waves $N(\mathbf{k})$, where $\mathbf{k} = (k_1 = k_x, k_2 = k_y)$ is the two-dimensional vector. In the dynamical sublayer of the stratified friction layer (or throughout the SAL for a neutral stratification), the mean wind velocity $U(z)$ is described by the logarithmic law

$$U(z) = \frac{U_*}{\kappa}\ln\frac{z}{z_0}, \tag{2.1}$$

where $\kappa \approx 0.4$ is the Karman constant. In the model of the logarithmic boundary layer, the parameters U_* and z_0 are independent. In the actual geophysical situation, the surface atmospheric layer is the bottom part of the planetary atmospheric boundary layer (PABL) with a given wind velocity U_g at its top boundary. Therefore, within the frameworks of the PABL model, the parameters U_* and z_0 in Eq. (2.1) are related to U_g and to the solution of the set of equations for the PABL. This relationship follows the Kazanskii–Monin geostrophic drag law for the PABL [9, 10]

$$U_* = \kappa U_g\left[\left(\ln\frac{U_*}{fz_0} - B\right)^2 + A^2\right]^{-1/2}, \tag{2.2}$$

where f is the Coriolis parameter and the values of the constants A and B depend on the stratification in the PABL and are determined from the numerical solution of the equations for the PABL.

To determine the relation between U_* and z_0 with U_g and $N(\mathbf{k})$ from Eq. (2.2), the total momentum flux τ_a in the friction layer is assumed as in [5] to be

$$\tau_a = -\rho_a U_*^2 = \tau_t + \tau_w. \tag{2.3}$$

Here,

$$\tau_t = -\rho_a U_{*t}^2 \tag{2.4}$$

is the momentum flux in the absence of waves (i.e., over a smooth underlying surface $z = 0$) and

$$\tau_w(U_{*t}, N) = -\rho_w g\int T^+(\mathbf{k}, U_{*t}, N)d\mathbf{k} \tag{2.5}$$

is the momentum flux to the waves from the atmosphere represented in terms of the corresponding spectrum of the momentum flux to the waves $T^+(\mathbf{k}, U_{*t}, N)$.

In the case of a smooth flow, the roughness parameter is determined from the known dimensionality considerations of the theory of wall turbulence [11]

$$z_{0\nu} = \alpha_\nu \nu/U_{*t}, \tag{2.6}$$

$$\alpha_\nu \approx 0.1 \tag{2.7}$$

($\nu \cong 0.15$ cm^2/s is the kinematic viscosity of the air). Therefore, the flux component in the SAL in Eq. (2.3), $\tau_t = -\rho_a U_{*t}^2$, which is unperturbed by the waves, can be directly determined from the corresponding detailing of the geostrophic drag law (2.2) for a smooth underlying surface and takes the form

$$\tau_t = -\rho_a U_{*t}^2(U_g, f, \nu, A, B). \tag{2.8}$$

By the substitution of (2.5) and (2.8) into (2.3), we obtain for U_*^2

$$U_*^2 = U_{*t}^2(U_g, f, \nu, A, B) + \tau_w(U_{*t}, N)/\rho_a. \tag{2.9}$$

Together with the geostrophic drag law (2.2), Eq. (2.9) represents a closed set of equations for determining the parameters of the surface atmospheric layer U_* and z_0 as functions of U_g, f, and N (for a given $T^+(\mathbf{k}, U_{*t}, N)$) in the form of (2.5)

$$\begin{aligned} U_* &= U_*(U_g, f, \nu, N, A, B),\\ z_0 &= z_0(U_g, f, \nu, N, A, B). \end{aligned} \tag{2.10}$$

To determine the spectrum of the momentum flux from the atmosphere to the waves using Eq. (2.5), we apply the general phenomenological transport equation for the spectrum of wind waves, which forms the wave unit of the coupled wind–wave model under consideration. This equation can be written for the spatial spectrum of wave forcing $N(\mathbf{k})$; then, the energy spectrum of waves $F(\mathbf{k})$ and the spectrum of wave momentum $M(\mathbf{k})$ may be expressed in terms of $N(\mathbf{k})$

$$F(\mathbf{k}) = \sigma(k)N(\mathbf{k}), \quad M(\mathbf{k}) = \mathbf{k}N(\mathbf{k}). \tag{2.11}$$

Here,

$$\sigma(k) = [gk\tanh(kh)]^{1/2} \tag{2.12}$$

is the dispersion relation for the surface gravity waves at the surface of the water column with a depth of h. For the normalization adopted, the energetic wave spectrum is normalized with respect to the dispersion $\langle\xi^2\rangle$ of the perturbations of the interface $\xi(\mathbf{x}, t)$

$$\int F(\mathbf{k})d\mathbf{k} = e = \langle \xi^2 \rangle. \tag{2.13}$$

For the same normalization, with an accuracy of the factor $\rho_w g$, the spectrum of the wave momentum is normalized with respect to the integral momentum

$$\int \mathbf{M}(\mathbf{k})d\mathbf{k} = \frac{\mathbf{m}}{\rho_w g}. \tag{2.14}$$

In the case unsteady and nonuniform over the horizontal $\mathbf{x} = (x_1 = x, x_2 = y)$, the evolution of the wave spectrum $N(\mathbf{k}, \mathbf{x}, t)$ is described by the general transport equation

$$\frac{dN}{dt} \equiv \frac{\partial N}{\partial t} + C(N, \sigma) = P, \tag{2.15}$$

where

$$C(N, \sigma) = \frac{\partial \sigma}{\partial k_x}\frac{\partial N}{\partial x} + \frac{\partial \sigma}{\partial k_x}\frac{\partial N}{\partial y} - \frac{\partial \sigma}{\partial x}\frac{\partial N}{\partial k_x} - \frac{\partial \sigma}{\partial y}\frac{\partial N}{\partial k_y} \tag{2.16}$$

is the convective term, and

$$P = P^0 + P^+ - P^- \tag{2.17}$$

is the source function that takes into account the nonlinear four-wave interactions in the spectrum of the gravity waves (P^0), their interaction with the atmosphere (P^+), and their small-scale dissipation (P^-). If the term P^0 is expressed in terms of N, and P^+ and P^- are represented in terms of N and U_{*t}, Eq. (2.15) can be used for the determination of the wave spectrum N in terms of U_{*t}

$$N = N(\mathbf{k}, \mathbf{x}, t, U_{*t}). \tag{2.18}$$

To determine the term responsible for the interaction of waves with wind P^+, we use the Miles' model linear in N [22]

$$P^+ = \beta\left(\frac{k_x U_{*t}}{\sigma(k)}\right)\sigma(k)N(\mathbf{k}) \tag{2.19}$$

and the empirical parameterization of the dimensionless coefficient of interaction of waves with wind according to the data obtained by Snyder et al. [25]

$$\beta(k_x U_{*t}/\sigma(k)) = \begin{cases} 0, & \text{at} \quad (\dots) < 0, \\ c_e \dfrac{\rho_a}{\rho_w}\left(c_d \dfrac{k_x U_{*t}}{\sigma(k)} - 1\right), & \text{at} \quad (\dots) \geq 0 \end{cases} \tag{2.20}$$

$$(c_e = 0.25,\ c_d = 28).$$

The form of the equation for P^+ allows us to write the corresponding details of the general Eq. (2.5) for the momentum flux from the atmosphere to the waves T^+, which should be substituted in the right-hand side of Eq. (2.9)

$$T^+(\mathbf{k}, U_{*t}, \sigma(k), N) = (k_x U_{*t}/\sigma(k))\sigma(k)k_x N(\mathbf{k}). \tag{2.21}$$

The empirical parameterization (2.20) of the coefficient of the wind–wave interaction β was obtained for the large-scale components of the wind waves with the dimensionless wavenumbers $\tilde{k} = kU_*^2/g \leq \tilde{k}_e \approx 10^{-2}$. In the numerical modeling of the energy-containing components of the wind waves for the field conditions from the transport equation (2.15), the small-scale components usually fall in the subgrid domain. Therefore, when we solve Eq. (2.15) for the wave spectrum, there is no need for the small-scale extrapolation of Eq. (2.20). Meanwhile, the contribution of the small-scale components to the total momentum flux from the atmosphere to the waves can be pronounced. Therefore, it should be taken into account in the atmospheric unit of the model in the calculations of τ_w from (2.5) and (2.21). For this purpose, we use an empirical parameterization of $\beta(k_x U_{*t}/\sigma(k))$ that is equivalent to that obtained by Plant [24] in the range of the dimensionless wavenumbers $10^{-2} < \tilde{k} < 10$

$$\beta\left(\frac{k_x U_{*t}}{\sigma(k)}\right) = a_s \frac{\rho_a}{\rho_w}\frac{k_x U_{*t}^2}{\sigma(k)}, \quad a_s \approx 30. \tag{2.22}$$

In the subgrid domain, the spectrum $N(\mathbf{k})$ in (2.22) is parameterized by the exponential "blocking" spec-

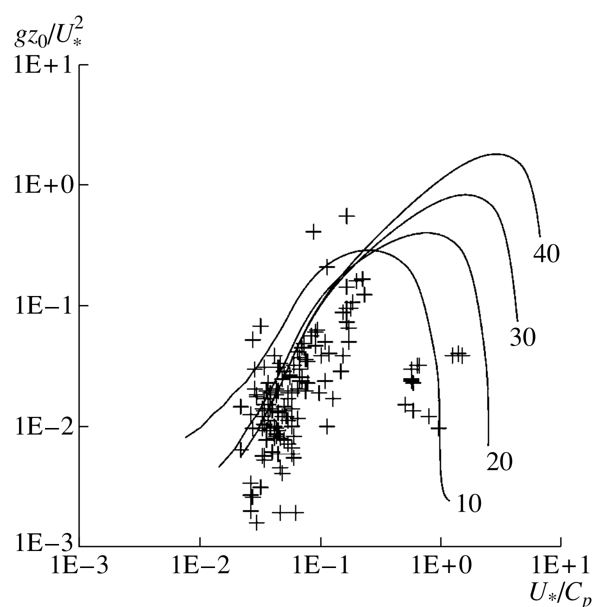

Fig. 1. Dimensionless roughness parameter as a function of the inverse wave age (U^*/C_p, where C_p is the wave phase velocity). The crosses mark the measurement data, the curves represent the results of the calculations with the model, and the numerals near the curves represent the velocities of the geostrophic wind.

trums (3.17) and (3.18). The substitution of them into (2.21) in view of Eq. (2.22) for β results in the short-wave divergence of the integral equation (2.5) for τ_w. To eliminate this divergence, we restrict the domain of the integral (2.5) to the wavenumber

$$k_0 = \alpha_0 U_{*t}/\nu, \quad \alpha_0 \cong 10^{-2}. \tag{2.23}$$

This restriction filters the small-scale components of the wind waves in the viscous sublayer ($h_\nu \cong 5\nu/U_{*t}$ in thickness), which make no contribution to the aerodynamical drag of the wavy surface.

Figure 1 shows the plots of the dimensionless roughness parameter against the inverse wave age obtained from the data of field observations [18] and calculated from the model described above. As can be seen, the plots are pronouncedly dependent on an external parameter; in the present experiment, it was the geostrophic wind velocity.

3. WAVE UNIT

If we use Eq. (2.19) for P^+, it is necessary to determine P^0 and P^- in the transport equations (2.15)–(2.17) for $N(\mathbf{k})$. The term describing the nonlinear four-wave interactions P^0 was obtained by Hasselmann [19] from the initial dynamical equations for free gravity waves for a horizontal homogeneous layer as

$$\begin{aligned} P^0(\mathbf{k}) &= 4\pi\int d\mathbf{k}_1\int d\mathbf{k}_2\int d\mathbf{k}_3 T^2(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\ &\times N(\mathbf{k}_1)N(\mathbf{k}_2)N(\mathbf{k}_3)\left[\frac{1}{N(\mathbf{k})} + \frac{1}{N(\mathbf{k}_1)} - \frac{1}{N(\mathbf{k}_2)}\right. \\ &\left. - \frac{1}{N(\mathbf{k}_3)}\right]\delta(\sigma(\mathbf{k}) + \sigma(\mathbf{k}_1) - \sigma(\mathbf{k}_2) - \sigma(\mathbf{k}_3)) \\ &\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3), \end{aligned} \tag{3.1}$$

where $\mathbf{k}$, $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the vectors.

Because Eq. (3.1) for P^0 is cumbersome, it is reasonable to use its simplified version obtained from the expansion of P^0 in series over the small parameter Δ [8]

$$\Delta = \Delta(k_x) = \kappa^2(k_x)/k_x^2 \ll 1, \tag{3.2}$$

where

$$\kappa^2(k_x) = \int k_y^2 N(\mathbf{k})dk_y/\bar{N}(k_x) \tag{3.3}$$

is the square of the effective cross section of the wave spectrum $N(\mathbf{k})$, and

$$\bar{N}(k_x) = \int N(\mathbf{k})dk_y \tag{3.4}$$

is the initial two-dimension wave spectrum $N(\mathbf{k})$ integrated over the transverse coordinate k_y. As was shown in [4], the parameter (3.2) can be expressed in terms of the general cosine parameterization of the function of the angular energy distribution

$$\begin{gathered} -\varphi(\omega, \theta) = A(n)\cos^{n(\omega)}(\theta), \text{ for} \\ -\pi/2 < \theta < \pi/2 \text{ and} \\ \varphi(\omega, \theta) = 0, \text{ for the remainder } \theta \text{ values,} \end{gathered} \tag{3.5}$$

where $A(n) = \Gamma(n+1)/2^n\Gamma^2((n+1)/2)$ and Γ is the gamma function. It can be shown that $\Delta(k_x)$ and $A(n)$ are correlated

$$A(n(k_x)) = [2\arctan(3\Delta(k_x))^{1/2}]^{-1}. \tag{3.6}$$

According to all the data of field observations [1, 4, 13], the power n in Eq. (3.5) ranges from 2 to 12, which corresponds to the range of variation of the parameter Δ from 0.04 to 0.3, so that the parameter Δ for wind waves can be really considered as a small one.

For the narrow directional approximation, instead of the initial two-dimensional wave spectrum $N(\mathbf{k})$, we seek its two integral parameters: $N(k_x)$ and $\Delta(k_x)$. Integrating the initial kinetic equation (2.15) for $N(\mathbf{k})$ over k_y, we eventually obtain its analog for $N(k_x)$ (see [3] for details)

$$\frac{d\bar{N}}{dt} = \bar{P}^0(k_x) + \bar{P}^+(k_x) - \bar{P}^-(k_x). \tag{3.7}$$

Here,

$$\begin{gathered} \bar{P}^0(k_x) = \int P^0(\mathbf{k})dk_y = 64\pi c_\Delta \frac{\partial}{\partial k_x} \\ \times\left\{\ln\left(\frac{1}{\Delta(k_x)}\right)\frac{\partial}{\partial k_x}[\Delta(k_x)k_x^{19/2}\varphi_1(k_x h)\bar{N}^3(k_x)]\right\}, \\ c_\Delta \approx 10^{-2}. \end{gathered} \tag{3.8}$$

In Eq. (3.8) $\varphi_1(k_x h)$ is a universal function that allows for the finiteness of the depth h (its explicit form is presented in [7]). Correct to the terms of the order of Δ, the forcing term $\bar{P}^+(k_x)$ in view of Eqs. (2.19) and (2.20) has the form

$$\bar{P}^+(k_x) = \int P^+(\mathbf{k})dk_y = \beta\left(\frac{k_x U_{*t}}{\sigma(k_x)}\right)\sigma(k_x)\bar{N}(k_x). \tag{3.9}$$

The dissipation term

$$\bar{P}^-(k_x) = \int P^-(\mathbf{k})dk_y \tag{3.10}$$

remains undetermined.

An additional equation for Δ can be obtained from Eq. (2.15) after its multiplying by k_y^2 and subsequent integrating over k_y

$$\frac{d}{dt}(\Delta\bar{N}) = \bar{R}^0(k_x) + \bar{R}^+(k_x) - \bar{R}^-(k_x). \tag{3.11}$$

Here,

$$\bar{R}^0(k_x) = \int k_y^2 P^0(\mathbf{k}) dk_y$$
$$= 64\pi c_\Delta \ln\left[\frac{1}{\Delta(k_x)}\right]\Delta(k_x) k_x^{15/2} \bar{N}^3(k_x)\varphi_2(k_x h), \quad (3.12)$$

$$\bar{R}^+(k_x) = \int k_y^2 P^+(\mathbf{k}) dk_y$$
$$= \beta\left(\frac{k_x U_{*t}}{\sigma(k_x)}\right)\sigma(k_x)\Delta(k_x)\bar{N}(k_x), \quad (3.13)$$

$$\bar{R}^-(k_x) = \int k_y^2 P^-(\mathbf{k}) dk_y. \quad (3.14)$$

In Eqs. (3.9) and (3.13), β is also determined from Eq. (2.20), and $\varphi_2(k_x h)$ in Eq. (3.12) is a universal function (its explicit form is also presented in [7]).

In order to provide the final details of the equation set (3.7)–(3.14) for $\bar{N}$ and Δ, we should determine the dissipative terms $\bar{P}^-$ and $\bar{R}^-$ in the right-hand sides of Eqs. (3.7)–(3.11). For this purpose, in our model for the calculation of wind waves, we use a representation of the blocking small-scale interval in the wind wave spectrum proposed in [6], which is a generalization of the known hypothesis on the saturation interval in the wind wave spectrum advanced by Phillips [15]. With the use of this representation, there is no need to a priori determine the approximation formulas for P^- (which is typical of all the present models for the prediction of wind waves). This term and the form of the wave spectrum in this scale range can be determined from P^+.

Using the Phillips' hypothesis on the saturation interval in the wind wave spectra in terms of $\bar{P}^-$, $\bar{P}^+$, and $\bar{P}^0$ in Eq. (3.7) for $\bar{N}$, let us assume that, in this interval, the interaction of the waves with the wind is blocked so that the total energy flux from the wind to the waves is balanced by its dissipation due to the small-scale breaking of wave crests, and the wind wave spectrum in this scale range $k_x > k_b$ becomes stationary. In terms of the kinetic equation (3.7), the first of these assumptions allows us to determine the dissipative term from the forcing term

$$\bar{P}^-(k_x) = \bar{P}^+(k_x), \quad \text{for} \quad k_x > k_b. \quad (3.15)$$

The second of these assumptions in the interval $k_x > k_b$ reduces Eq. (3.7), in view of Eq. (3.15), to the equation

$$\bar{P}^0(k_x) = 64 c_\Delta \pi \frac{\partial}{\partial k_x}\left[\ln\left[\frac{1}{\Delta(k_x)}\right]\frac{\partial}{\partial k_x}\right.$$
$$\left.\times[\Delta(k_x) k_x^{19/2}\bar{N}^3(k_x)\varphi_1(k_x h)]\right] = 0. \quad (3.16)$$

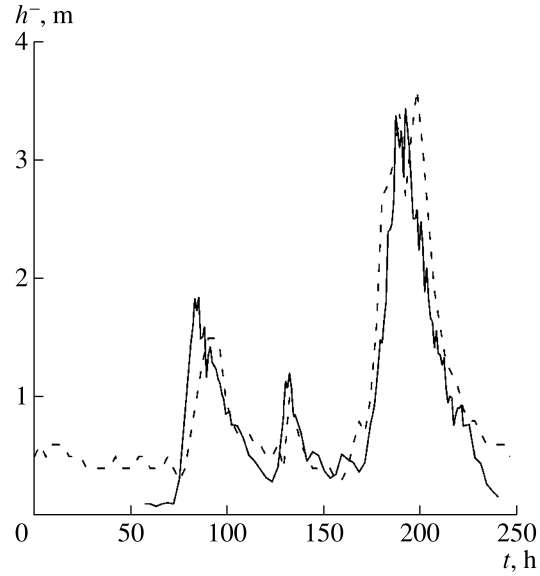

Fig. 2. Calculated (dashed line) and measured (solid line) mean wave height ($\bar{h}$, m) history during a storm (t, h) from April 4 to 14 of 1997 in the region of Gelendzhik in the Black Sea. The moment $t = 0$ corresponds to 3:00 a.m. on April 4, 1997.

A direct substitution into Eq. (3.16) shows that, at $kh \gg 1$, this assumption is valid for

$$\bar{N}(k_x) = \bar{N}_b(k_x) = c_b g^{-5/6} U_*^{2/3} \Delta^{-1/3}(k_x) k_x^{-19/6}, \quad (3.17)$$

where c_b is the dimensionless coefficient of proportionality. The spectrum $\bar{N}_b(k_x)$, which satisfies the stationary condition (3.16), will be referred to as the blocking spectrum. It differs from the corresponding Phillips' saturation spectrum

$$\bar{N}_p(k_x) = B g^{-1/2} k_x^{-7/2}, \quad B \cong 5 \times 10^{-3}. \quad (3.18)$$

A more detailed consideration [6] shows that the spectrum (3.18) can also be considered as a strongly nonlinear case (for $ke^{1/2} \gg 1$) of the blocking spectra determined by Eq. (3.15). It should be noted, however, that Eq. (3.17) and the initial four-wave kinetic integral (3.1) as well are valid for a weakly nonlinear case (for $ke^{1/2} \gg 1$), which can be applied to the actual wind waves only if $k < 10k_m$ [6].

For the calculations with the proposed narrow directional model for predicting wind waves, a very important problem is to choose the bottom boundary k_b of the blocking interval. This is due to the fact that, for $k_x = k_b$, the unsteady spectrum of wind waves $\bar{N}$ calculated with the use of the model should be sewed together with the stationary blocking spectra (3.17). The unknown value of k_b can be determined from the integral conservation law of wave forcing

$$\int_0^{k_b} \bar{P}^0(k_x)dk_x = 0. \quad (3.19)$$

The appropriate numerical calculations show that, for fully developed wind waves,

$$k_b = \alpha_b k_m, \quad \alpha_b \cong 4. \quad (3.20)$$

The model presented was tested by the data of the field observations obtained with a WAVERIDER moored wavegauge in the region of Gelendzhik (the Caucasian coast of the Black Sea). The calculations were performed for a series of storm events that occurred from April 4 to 14, 1997. The results of the comparison are presented in Fig. 2. As can be seen from the figure, the model simulates well the qualitative and quantitative characteristics of the storm events that occurred in the region of Gelendzhik in the study period. The error of the simulation of the mean wave height does not exceed 20 cm (about 10%). The moments when the maximums come are also well simulated.

4. SEA DYNAMICS UNIT

The base of the sea dynamics unit can be represented by any rather complete semiempirical k–ε model of sea currents that requires the values of the momentum and energy fluxes from the wind and waves to the drift boundary layer [2, 17]. An innovation in the method proposed is the use of a special procedure for coupling the units of atmospheric and sea dynamics by imposing the boundary conditions on the fluxes of appropriate quantities.

Let us consider a nonhydrostatic k–ε model for turbulent sea currents used in the numerical calculations. The model is described by the equation set [2]

$$\frac{du}{dt} + nwu - \hat{f}v + \tilde{f}w = \frac{\partial}{\partial z}\nu_u\frac{\partial u}{\partial z} + F_1,$$

$$\frac{\partial v}{\partial t} + nwv + \hat{f}u = \frac{\partial}{\partial z}\nu_u\frac{\partial v}{\partial z} + F_2,$$

$$\frac{dw}{dt} - n(u^2+v^2) - \tilde{f}u = \frac{\partial}{\partial z}\nu_u\frac{\partial w}{\partial z} + F_3,$$

$$mn\left[\frac{\partial}{\partial\lambda}\left(\frac{u}{n}\right) + \frac{\partial}{\partial\phi}\left(\frac{v}{m}\right)\right] + \frac{\partial w}{\partial z} = 0,$$

$$\frac{dT}{dt} = F_T, \quad (4.1)$$

$$\frac{dS}{dt} = F_S,$$

$$\frac{dk}{dt} = \gamma_1\frac{k^2}{\varepsilon} - \varepsilon + \frac{\partial}{\partial z}\frac{\nu_u}{\sigma_k}\frac{\partial k}{\partial z} + F_k,$$

$$\frac{d\varepsilon}{dt} = c_1 k\gamma_2 - c_2\frac{\varepsilon^2}{k} + \frac{\partial}{\partial z}\frac{\nu_u}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial z} + F_\varepsilon,$$

$$\rho = \rho(T, S, p).$$

The right-hand sides of set (4.1), denoted as F_1, F_2, …, F_ε, include all the additional terms for the appropriate equations. The notation used in equation set (4.1) can be found in [2]. Below, we present only those of them that are necessary for the present study: λ is the longitude, ϕ is the latitude, z is the vertical coordinate directed downward, $\hat{f} = 2\Omega\sin\phi$, $\tilde{f} = 2\Omega\cos\phi$, Ω is the angular velocity of the earth's rotation, k is the kinetic energy of turbulence, ε is the rate of dissipation of the turbulent kinetic energy,

$$\gamma_1 = c_\mu\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - \frac{1}{\sigma_T}\frac{\partial\rho}{\partial z}\right],$$

$$\gamma_2 = \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - \frac{c_3}{\sigma_T}\frac{\partial\rho}{\partial z}\right],$$

σ_t is the Prandtl–Schmidt number, $\nu_* = c_\mu k^2\varepsilon^{-1}$ is the vertical turbulent diffusivity, and c_μ, c_1, c_2, and c_3 are the constants. The $k - \varepsilon$ model equations are presented according to [17].

It should be noted that σ_t can be a constant or a function of the Richardson number Ri_t and determined, e.g., from the Munk–Andersen formula

$$\sigma_t = \begin{cases} \left(1 + \frac{10\mathrm{Ri}_t}{3}\right)^{3/2} / (1 + 10\mathrm{Ri}_t)^{1/2}, & \text{for} \quad \mathrm{Ri}_t \geq 0 \\ 1, & \text{for} \quad \mathrm{Ri}_t < 0. \end{cases} \quad (4.2)$$

The standard values of the constants are the following:

$$\sigma_k = 1, \quad \sigma_\varepsilon = 1.3, \quad c_\mu = 0.09, \quad c_1 = 1.44, \quad c_2 = 1.92, \quad c_3 = \begin{cases} 0, & \rho_z > 0 \\ 1, & \rho_z \leq 0. \end{cases} \quad (4.3)$$

At the top boundary of the sea, for $z = 0$, we impose the following boundary conditions

$$\nu_u\frac{\partial u}{\partial z} = -\frac{\tau_d}{\rho_0}\cos\alpha,$$

$$\nu_u \frac{\partial v}{\partial z} = -\frac{\tau_d}{\rho_0}\sin\alpha,$$
$$\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial k}{\partial z} = -\frac{q_d}{\rho_0}, \tag{4.4}$$
$$\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z} = -\frac{q_\varepsilon}{\rho_0}.$$

Here, τ_d and q_d are the fluxes of momentum and energy to the drift current from the wind and waves, and α is the angle between the direction of the wind and the x-axis (directed to the east). At the bottom boundary of the sea, we assume that all the fluxes vanish.

Let us then consider only the dynamical fluxes τ_d and q_d and determine the momentum flux τ_d in Eq. (4.4). The results obtained in Sections 2 and 3 show that two effects make a contributions to τ_d. They are, first, the presence of viscous friction at the water surface described by the term τ_t in Eq. (2.3) for τ_a and, second, the fact that the momentum flux τ_w from the atmosphere to the waves cannot be completely assimilated by the waves. It can be written as

$$\tau_w = \tau_w^+ + \tau_w^-, \tag{4.5}$$

where

$$\tau_w^+ = -\rho_w g\int_0^{k_b} T^+(\mathbf{k})d\mathbf{k}$$
$$= -\rho_w g\int_0^{k_b}\beta(k_x, U_{*t})\sigma(k_x)k_x\bar{N}(k_x)dk_x \tag{4.6}$$

is the long-wave component of τ_w spent on the wave growth, and

$$\tau_w^- = -\rho_w g\int_{k_b}^{k_0} T^+(\mathbf{k})d\mathbf{k}$$
$$= -\rho_w g\int_{k_b}^{k_0}\beta(k_x, U_{*t})\sigma(k_x)k_x\bar{N}_b(k_x)dk_x \tag{4.7}$$

is the short-wave component, which is supplied to the waves from the wind in the blocking interval $k > k_b$, but it is not assimilated by the waves and enters the drift current causing the breaking down of the wave crests. Therefore, τ_d should satisfy the following equation

$$\tau_d = \tau_w = \tau_w^+ + \tau_w^- < \tau_t + \tau_w. \tag{4.8}$$

This equation shows that, as the wind waves develop, the condition of the constancy of the momentum flux in the vicinity of the air–sea interface is not satisfied. Therefore, the use of the general drift models, which use the condition $\tau_d = \tau_a$, results in an overestimation of the value of the momentum flux to the drift current.

Let us now determine τ_w^- from Eq. (4.7) ignoring the relatively small difference between the powers n_b = 19/6 and n_p = 7/2 in the weakly nonlinear blocking spectrum (3.17) and the strongly nonlinear Phillips spectrum (3.18) ($(n_p - n_b)/n_p = 2/21 \approx 0.1$) and assuming $\bar{N}_b(k_x) = \bar{N}_p(k_x)$ in Eq. (4.7). Let us then use the short-wave parameterization (2.22) for $\beta(k_x U_{*t})$ throughout the blocking interval. (These two simplifications somewhat overestimate the value obtained for τ_w^-.) Within the limits of the integration k_0, k_b in Eq. (4.7), we will use Eqs. (3.20) and (2.23). The first equation is valid for the case of fully developed waves, and the second one is valid for the case of moderate wind velocities with $U_{*t} < 0.3$ m/s. Then,

$$\bar{\tau}_w = -A\rho_a U_{*t}^2, \quad A = Ba_s\ln\left(\frac{a_0}{a_b}\frac{U_{*t}}{k_m\nu}\right) \approx 0.9 \tag{4.9}$$

for U_{*t} = 0.3 m/s, $\tilde{k}_m = k_m U_{*t}^2/g = 10^{-3}$, $a_0 = 10^{-2}$, $a_b = 4$, $B = 5\times10^{-3}$, $a_s = 30$.

As can be seen from this estimation, the wind waves are good catalysts of the drift currents—they double the value of the momentum flux to the drift current in comparison to the case when the waves are absent, i.e., when $\tau_d = \tau_t = -\rho_a U_{*t}^2$. Finally, using (4.8) and (4.9), we can write the following equation for τ_d in (4.4)

$$\tau_d = \tau_t + \bar{\tau}_w = \rho_a U_{*t}^2\left[1 + Ba_s\ln\left(\frac{a_0}{a_b}\frac{U_{*t}}{k_m\nu}\right)\right]. \tag{4.10}$$

From Eq. (4.10), it follows that, for U_{*t} = const, the momentum flux to the drift current increases as the waves develop, although the correlation is weak. When k_m diminishes by a factor of 10 (which corresponds to the variability in the field conditions), τ_w^- grows approximately twofold and τ_d increases by a factor of one and a half.

Now, let us calculate the energy flux q_d that is presented in the boundary condition (4.4) for the equation of the turbulent energy balance. With regard to the energy flux from the small-scale wave breaking $\bar{q}_w$, we have

$$q_d = q_t + \bar{q}_w, \tag{4.11}$$

where

$$\bar{q}_w = -\rho_w g\int_{k_b}^{\infty}\beta(k_x, U_{*t})\sigma^2(k_x)N(k_x)dk_x \tag{4.12}$$

(in contrast to $\bar{q}_w$, when we determine $\bar{\tau}_w$, there is no problem with the logarithmic divergence as $k_x \to \infty$,

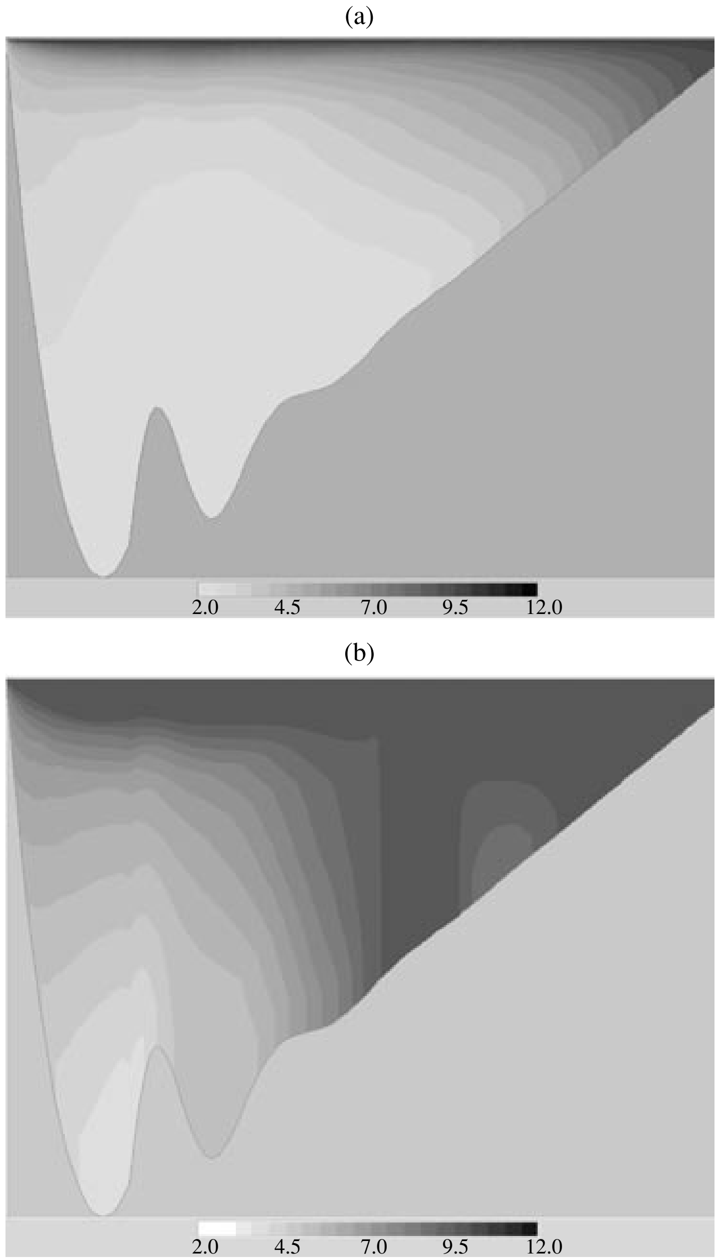

Fig. 3. Temperature in the west–east section along the academic basin after the 90-day calculations. (a) With no account for wind waves, and (b) with account for wind waves.

which we have to solve by cutting the integral (4.7) at $k_x = k_0$).

For the calculation of $\bar{q}_w$ from Eq. (4.12), we adopt the same simplifying assumptions as for the derivation of Eq. (4.9). Then

$$\bar{q}_w = 2Ba_S\rho_a U_{*t}^2\left(\frac{g}{k_b}\right)^{1/2} \approx 0.3\rho_a U_{*t}^2\left(\frac{g}{k_b}\right)^{1/2}. \quad (4.13)$$

As was mentioned above, for fully developed waves with $c_m = (g/k_m)^{1/2} \cong U_a$, the results of the calculations show that $k_b \cong 4k_m$. Then, $\bar{q}_w \cong 0.15\rho_a U_{*t}^2 U_a$. On the other hand, the velocity of the surface drift is $U_0 \cong 0.03U_a \cong U_* \approx U_{*t}$, so that

$$q_t \cong \rho_a U_*^3 \cong 0.2\bar{q}_w. \quad (4.14)$$

Thus, at least for the case of fully developed waves, the inequality $q_w \gg q_t$ is valid. On the basis of this inequality, the general equation (4.11) for q_d can be written as

$$q_d \cong (1 + c_w)U_*^3, \quad c_w \cong 5. \quad (4.15)$$

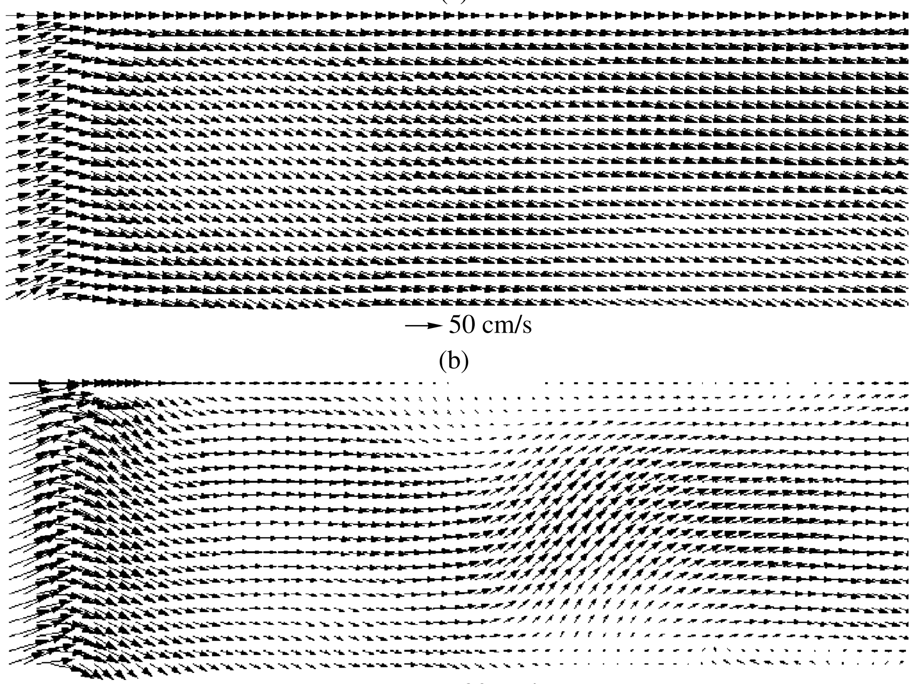

Fig. 4. Sea surface currents. (a) With no account for wind waves, and (b) with account for wind waves.

The relative value of the fluxes τ_d and q_d can be determined from the dimensionless equation $\mu = (\tau_d/\rho_a)^{3/2}/(q_d/\rho_a)$. From Eqs. (4.10) and (4.13), it follows that

$$\mu = \frac{\left[1 + Ba_s \ln\left(\frac{a_0}{a_s}\frac{U_{*t}}{k_m \nu}\right)\right]^{3/2}}{2Ba_s\left(\frac{g}{a_b k_m U_{*t}^2}\right)^{1/2}}. \qquad (4.16)$$

Using in (4.16) the same numerical values of the constants as in (4.9), we find that $\mu \cong 1.2$.

5. NUMERICAL EXPERIMENT ILLUSTRATING THE ROLE OF WIND WAVES IN THE FORMATION OF THE DYNAMICS OF THE SEA

The calculations were performed for a rectangular (academic) basin with a variable bottom relief [2]. The academic basin simulates the northern Baltic Sea that lies at the latitude of the Gulf of Finland from the coast of Sweden to the estuary of the Neva River. The basin is bounded by a closed coastal surface. The southern and northern boundaries of the basin are located at 60° and 61° N, while the western and eastern boundaries are at 19° and 30°40′ E. The bottom topography is described by an analytical function that simulates the characteristic features of the bottom in the Gulf of Finland. The maximum and minimum depths of the basin are 180 and 1 m, respectively.

The calculations were performed in a prognostic regime with the use of the nonhydrostatic σ-model of the sea dynamics [2]. As the boundary conditions at the surface, we used the mean climatic conditions characteristic of the Baltic Sea: a constant southwesterly wind with a velocity of 7 m/s and an air temperature of 10°C. The mesh sizes over the longitude and latitude were 5 miles, and the step of the integration of the model in time was 0.25 h. At the initial moment, motion is absent and the temperature field is given by [21]

$$T = T_0 + \Theta(\sigma)[T_H - T_0], \quad T_0 = 3°\text{C}, \quad T_H = 1°\text{C},$$

$$\Theta(\sigma) = 1 - 4(1-\sigma)^3 + 3(1-\sigma)^4,$$

$$\sigma = z/H(\lambda, \phi).$$

We performed two 90-day runs with and without accounting for the effect of the wind waves. The difference between the calculations was manifested in the different values of the coefficient in the boundary condition (4.4) at the surface for the flux of the turbulent kinetic energy $q_d = (1 + c_w)U_*^3$ [16]. With no regard for the effect of the wind waves $c_w = 0$, whereas, with account for it, according to the theoretical estimation (4.14) presented above, $c_w = 5$.

Figures 3–5 show the characteristics of the temperature and currents for the first and second runs. As can

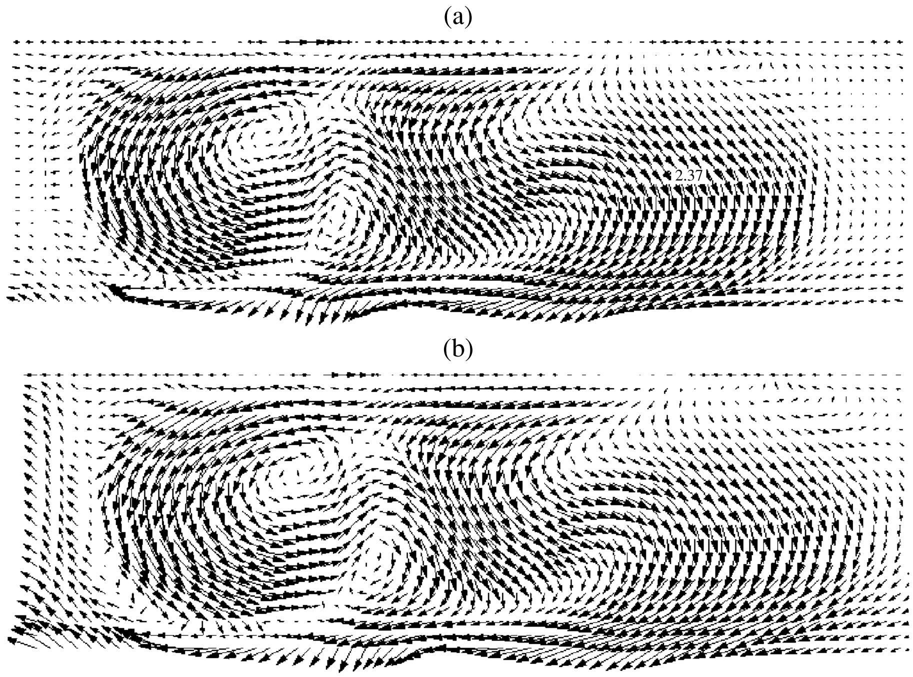

Fig. 5. Near-bottom currents. (a) With no account for wind waves, and (b) with account for wind waves.

be seen from the figures, the presence of wind waves clearly affects the dynamics of the currents and temperature field. The contribution of wind waves is particularly pronounced in the eastern shallow-water part of the academic basin and in the surface layer. In the bottom layer, the patterns of circulation are similar, although there are some differences near the eastern and western boundaries. The calculations show that the effect of wind waves should be taken into account in the analysis and prediction of the hydrodynamical and thermodynamical processes in the upper active sea layer.

6. CONCLUSIONS

In the present study, we considered the general method for coupling the model units describing the atmospheric circulation and the dynamics of the ocean at the sea–air interface with the use of an additional wave unit. The wave dynamics unit is based on the transport equation for the spectrum of wind waves in the strictly directional approximation and on a special procedure for separation of wave perturbations in the boundary layer of the air and water. The wave dynamics unit is used for determining the spectrum of the wind waves and coupling the models of turbulent boundary layers in the air and water. Using this unit, we calculated various characteristics of the appropriate fluxes required for imposing boundary conditions at the interface. As a result, we obtained a closed self-adjusted description of the wind, waves, and sea currents with account for the adjustment of all these processes.

We described the model of wind wave dynamics in the strictly directional approximation. The model was tested using the data of field observations of a series of storm events in the region of Gelendzhik (the Black Sea).

We presented a theoretical estimation of the contribution of wind waves to the fluxes of momentum and turbulent kinetic energy at the sea surface.

With the use of a nonhydrostatic numerical model of the thermohaline dynamics of the sea provided with the unit of the $k - \varepsilon$ parameterization of turbulence, we assessed the effect of wind waves on the dynamics of the sea. The experiment was carried out for an idealized domain that simulated the conditions in the Gulf of Finland. The calculations show a pronounced contribution of wind waves to the formation of the dynamics of the upper sea layer.

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