An investigation of the structure of perturbation coefficients for compensation of fiber nonlinear distortions

TATIANA SHELOPUT[1], NIKOLAY ZAMARASHKIN[1,2], DMITRY ZHELTKOV[1], ILYA KOSOLAPOV[1,3], ROMAN DYACHENKO[4] ✉ sheloput@phystech.edu
[1]INSTITUTE OF NUMERICAL MATHEMATICS RAS, RUSSIAN FEDERATION, [2]LOMONOSOV MOSCOW STATE UNIVERSITY, RUSSIAN FEDERATION, [3]MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY, RUSSIAN FEDERATION, [4]SKOLKOVO INSTITUTE OF SCIENCE AND TECHNOLOGY, RUSSIAN FEDERATION

1. Introduction

- Fiber optic nonlinearity is the main limitation for increasing link distance and spectrum efficiency.
- There are some approaches for analyzing the fiber nonlinearity, for example, numerical approach (digital back propagation, DBP), Volterra analysis, perturbation analysis [1].

The main goal of this work is to modify methods for computing coefficients of PBM model in order to improve the optical fiber transmission quality. It is shown that computing the matrix of PBM coefficients in the form of a low-rank approximation allows to obtain coefficients having a certain structure and significantly reduce the number of multiplications.

2. Signal transmission model

The Manakov equation that governs the propagation of a polarized optical pulse in a dispersive fiber medium can be written as [1]

$$\frac{\partial}{\partial z}\boldsymbol{u}(t,z)+j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\boldsymbol{u}(t,z)+\frac{\alpha-g(z)}{2}\boldsymbol{u}(t,z)=$$

$$=j\frac{8}{9}\gamma|\boldsymbol{u}(t,z)|^2\boldsymbol{u}(t,z),$$

where $\boldsymbol{u}(t,z) = [u_x(t,z),\ u_y(t,z)]^T$ is the slowly varying envelope of the electrical field in two orthogonal polarizations, α - fiber loss coefficient, $g(z)$ simulates signal amplification, β_2 characterizes chromatic dispersion, γ is supposed to be a small coefficient that characterizes Kerr nonlinearity.

3. PBM model

Let us consider the input signal as

$$\boldsymbol{u}(t,0)=\sum_{k=-\infty}^{+\infty}\boldsymbol{x}_k b_0(t-kT),\quad \boldsymbol{x}_k=\begin{bmatrix}x_X(k)\\ x_Y(k)\end{bmatrix}$$

where $b_0(t)$ means an input pulse. nonlinear interactions yield a perturbation of the sampled at $t=kT$ output signal:

$$\boldsymbol{y}_k\approx\boldsymbol{x}_k+\boldsymbol{\Delta}_k,\quad \boldsymbol{\Delta}_k=[\delta_X(k),\delta_Y(k)]^T,$$

$$\delta(k)=\sum_{m,n=-\infty}^{+\infty}(x(k+n)x^*(k+n+m)x(k+m)+$$

$$+x_o(k+n)x_o^*(k+n+m)x(k+m))a_{m,n},\quad (1)$$

where δ means δ_X or δ_Y and the index "o" means the other polarization.
Problem: find the coefficients $a_{m,n}$ so that the number of multiplications while computing $\delta_{X|Y}(k)$ is low and $\boldsymbol{y}_k-\boldsymbol{\Delta}_k$ is close to $\boldsymbol{x}_k$.

Acknowledgments

The work was supported by the Huawei Technologies Co. Ltd.

4. Low-rank approximation

Let us suppose that the coefficients of the PBM model satisfy the following condition:

$$a_{m,n}=a_{-m,-n}=-a^*_{m,-n}=-a^*_{-m,n},$$

Assume that the quadrants of the coef. matrix are approximated by small rank matrices:

$$a_{m,n}\approx\sum_{r=1}^{R}U^{(1)}_{m,r}V^{(1)}_{n,r}.$$

Let the quadrant of the whole matrix satisfy the condition $A=A^T$. It is known [2] that for a matrix $A\in\mathbb{C}^{N\times N}$ satisfying $A=A^T$ exists a unitary matrix Q such that

$$A=Q\Sigma Q^T,$$

where $\Sigma = diag(\sigma_1,\dots,\sigma_n)$ with real non-negative σ_i. This special singular value decomposition is called SSVD.

References

[1] Bakhshali, Ali *"Nonlinearity compensation for next generation coherent optical fiber communication systems."* Diss. Queen's University (Canada), 2017.
[2] Bunse-Gerstner, Angelika, and William B. Gragg. *"Singular value decompositions of complex symmetric matrices."* Journal of Computational and Applied Mathematics 21.1 (1988): 41-54.

5. Iterative algorithm

We propose the following projective algorithm SQA-ALS (for simplicity single polarization case described):

1. Let U and V be decomposition factors obtained from the previous iteration, $A=UV^T$. Fix factor V and perform one ALS step: find U^{new}, α, minimizing $\|\boldsymbol{y}-\boldsymbol{x}\|^2$.

2. $A=U_{new}V^T$. Compute SSVD of $\tilde{A}=(A+A^T)/2$. Using SSVD, obtain an approximation to $\tilde{A}$ of rank R by setting the singular numbers $\sigma_{R+1},\dots,\sigma_N$ to zero. This approximation is denoted by $A_{1/2}$. From SSVD factor $U_{1/2}$ such that $A_{1/2}=U_{1/2}U^T_{1/2}$ is obtained.

3. Perform one ALS step: varying factor V in $A_{1/2}=U_{1/2}V^T$ and the linear term, find V_{new}, α, minimizing $\|\boldsymbol{y}-\boldsymbol{x}\|^2$.

4. $A=U_{1/2}V^T_{new}$. Compute SSVD of $\tilde{A}=(A+A^T)/2$. Using SSVD, obtain an approximation to $\tilde{A}$ of rank R by setting the singular numbers $\sigma_{R+1},\dots,\sigma_N$ to zero. This approximation is denoted by A_{fin}. From SSVD factor U_{fin} such that $A_{fin}=U_{fin}U^T_{fin}$ is obtained.

5. Check if the stopping criterion is satisfied. If not, repeat steps 1-5.

Note that true inputs are unknown on the output side, so $\boldsymbol{y}$ is used insted of $\boldsymbol{x}$ to compute triplets in (1).

7. Results

		full PBM		SQA-ALS	
(N, M)	R	NMSE	BER	NMSE	BER
before PBM		-11.20	1.97e-02		
(32, 32)	2			-13.04	6.05e-03
(32, 32)	3	-13.161	5.56e-03	-13.10	5.93e-03
(32, 32)	4			-13.12	5.82e-03
(64, 64)	2			-13.57	3.84e-03
(64, 64)	3	-14.111	2.52e-03	-13.77	3.37e-03
(64, 64)	4			-13.86	3.15e-03
(128, 128)	2			-13.88	2.9e-03
(128, 128)	3	-15.307	0.79e-03	-14.24	2.1e-03
(128, 128)	4			-14.44	1.8e-03
(256, 256)	2	-16.586	0.22e-03	-13.99	2.6e-03
(256, 256)	4			-14.74	1.3e-03

Table 1. The error of the ALS-based corrected solution depending on M, N and rank (X polarization), compared to the full PBM model (without the assumption of a small rank) result.

Result 1. Algorithm for computation coefficient matrix in a low-rank form was proposed.
Result 2. Using the Haar transform, efective piecewise constant approximation of coefficient matrix was proposed.
Result 3. Experiments with the full PBM model allowed to estimate the efficiency losses arising under the low rank assumption.

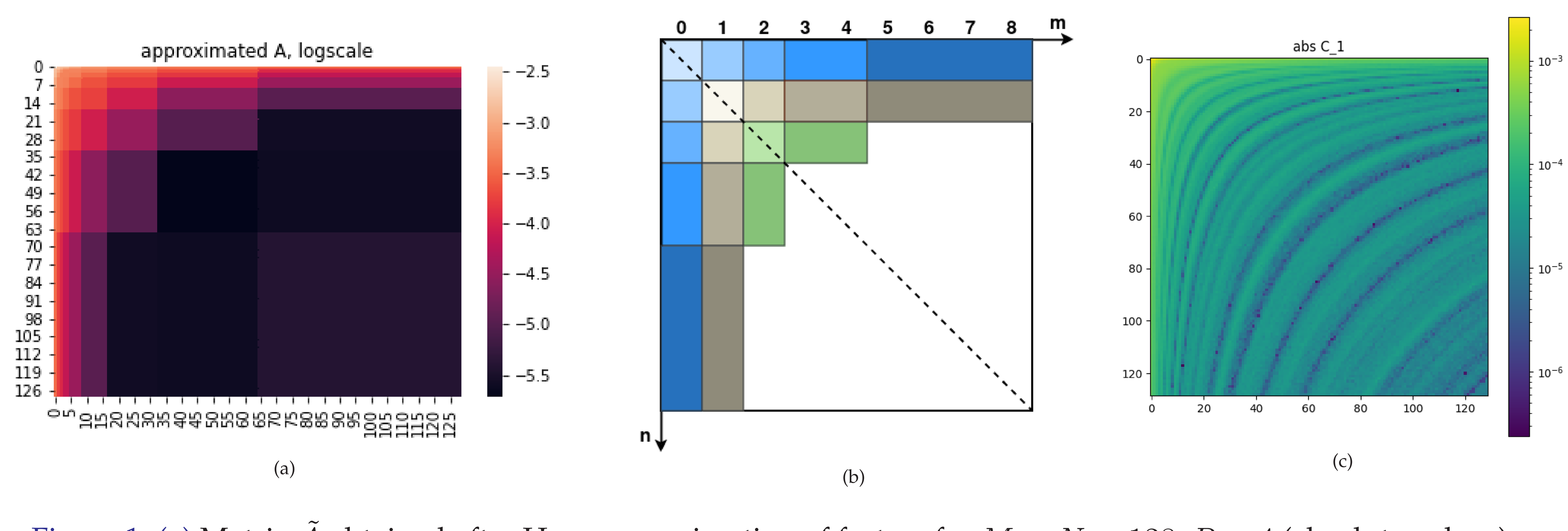

Figure 1. (a) Matrix $\tilde{A}$ obtained after Haar approximation of factors for $M=N=128$, $R=4$ (absolute values). (b) Schematic description of the position of the averaging blocks in the case $M=N=8$, $T=1$. Coefficients marked in white are not taken into account. (c) full PBM model result (absolute values).