Precoder calculation in MIMO systems using Tucker Decomposition

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Introduction

Precoding in Multiple-Input Multiple-Output (MIMO) systems is a signal processing technique applied at the transmitter side to improve the overall performance of the communication system. Among the possible benefits of using precoding are diversity gain, spatial multiplexing, and many others. The baseline scenario for precoding calculation requires singular vector decomposition (SVD) independently calculated on the channel matrix for each subcarrier, which is very complex. However, the same channel could be processed as a tensor utilizing information from all subcarriers. **The goal** of this research is to investigate Tucker decomposition for precoding calculation and analyze the physical properties of this decomposition.

System model

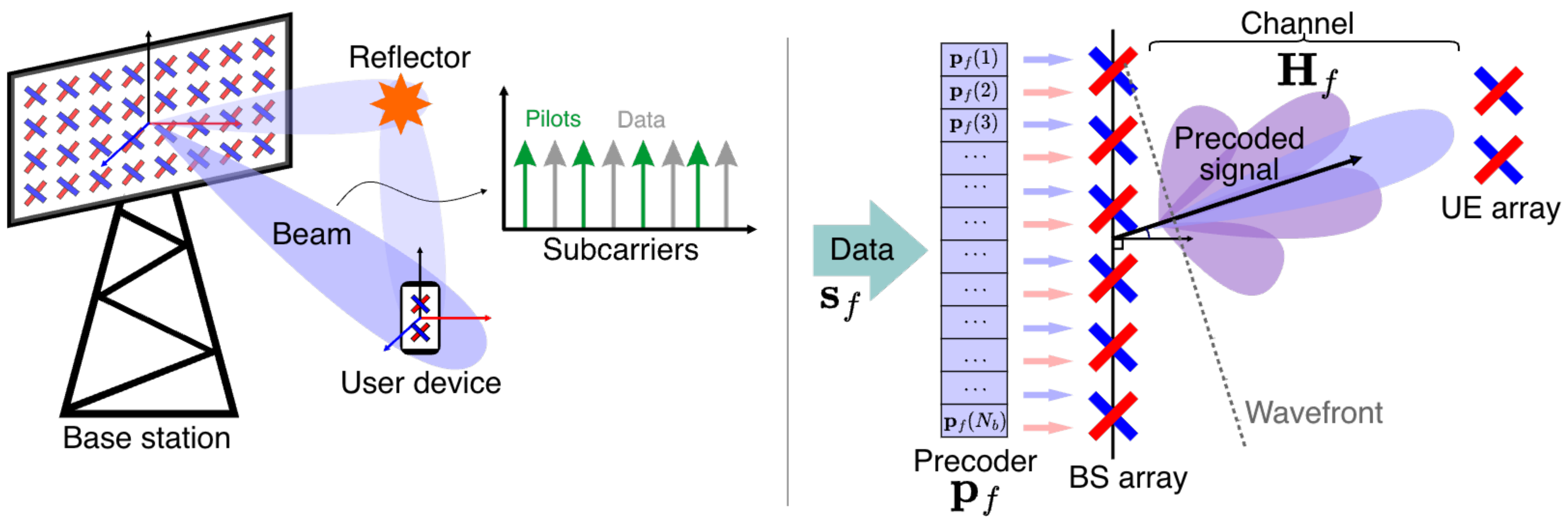

Figure 1. Left: Illustration of the signal propagation in the environment; Right: Illustration of the MIMO precoding principle

Consider a MIMO system with N_b antennas at base station (BS) and N_u antennas at the user equipment (UE). Let $\mathbf{H}_f \in \mathbb{C}^{N_b \times N_u}$ denote the channel between BS and UE of the fixed subcarrier $f \in \{0, ..., N_f\}$, where N_f is a total number of subcarriers used for transmission. Let the BS transmitted signal be $\mathbf{s}_f \in \mathbb{C}^{N_b \times 1}$. Then received signal $\mathbf{y}_f \in \mathbb{C}^{N_u \times 1}$ can be expressed by:

$$\mathbf{y}_f = \mathbf{H}_f \mathbf{P}_f \mathbf{s}_f + \mathbf{n} \tag{1}$$

Where $\mathbf{P}_f \in \mathbb{C}^{N_b \times N_b}$ denotes the precoder, $\mathbf{n} \in \mathbb{C}^{N_u \times 1}$ is the white Gaussian noise. Let us use singular value decomposition of the channel: $\mathbf{H}_f = \mathbf{U}_f \mathbf{\Sigma}_f \mathbf{V}_f^H$ where $\mathbf{U}_f \in \mathbb{C}^{N_u \times N_u}$, $\mathbf{V}_f \in \mathbb{C}^{N_b \times N_b}$ are unitary, representing left and right singular vectors; $\mathbf{\Sigma}_f \in \mathbb{R}^{N_u \times N_b}$ is a diagonal matrix containing real singular values. Often matrix $\mathbf{\Sigma}_f$ has $R < \min(N_b, N_u)$ non-zero singular values, so $R = \operatorname{rank}(\mathbf{H}_f)$.

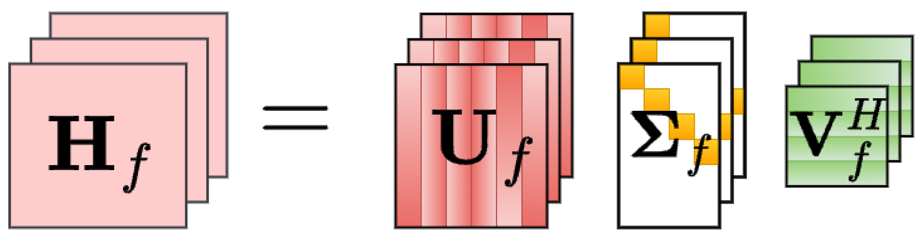

Figure 2. Illustration of singular value decomposition for each subcarrier channel matrix

Using precoder $\mathbf{P}_f = \mathbf{V}_f$ and equalizer $\mathbf{U}_f^H$ at the receiver, received signal can be represented as following:

$$\mathbf{U}_f^H \mathbf{y}_f = \mathbf{U}_f^H \mathbf{H}_f \mathbf{V}_f \mathbf{s}_f + \mathbf{U}_f^H \mathbf{n} = \mathbf{\Sigma}_f \mathbf{s}_f + \mathbf{U}_f^H \mathbf{n} \tag{2}$$

From the obtained result it could be noted that precoder $\mathbf{P}_f$ provides an opportunity to send transmitted signal $\mathbf{s}_f$ independently using each singular vector. This result could be used for many wireless channel applications. This technique is required to be calculated for each subcarrier f and for any moment of time, which makes this procedure complex. However, MIMO channel naturally can be represented as a tensor: $\mathcal{H} \in \mathbb{C}^{N_b \times N_u \times N_f}$, then classical methods for tensor decompositions could be used for precoder calculation, for example, Tucker decomposition can be derived as follows:

$$\mathcal{H} = \mathcal{C} \times_1 \mathbf{U}^{(b)} \times_2 \mathbf{U}^{(u)} \times_3 \mathbf{U}^{(f)} = \sum_{r_b=1}^{Rb} \sum_{r_u=1}^{R_u} \sum_{r_f=1}^{Rf} \mathcal{C}(r_b, r_u, r_f) \mathbf{U}^{(b)}(r_b) \circ \mathbf{U}^{(u)}(r_u) \circ \mathbf{U}^{(f)}(r_f) \tag{3}$$

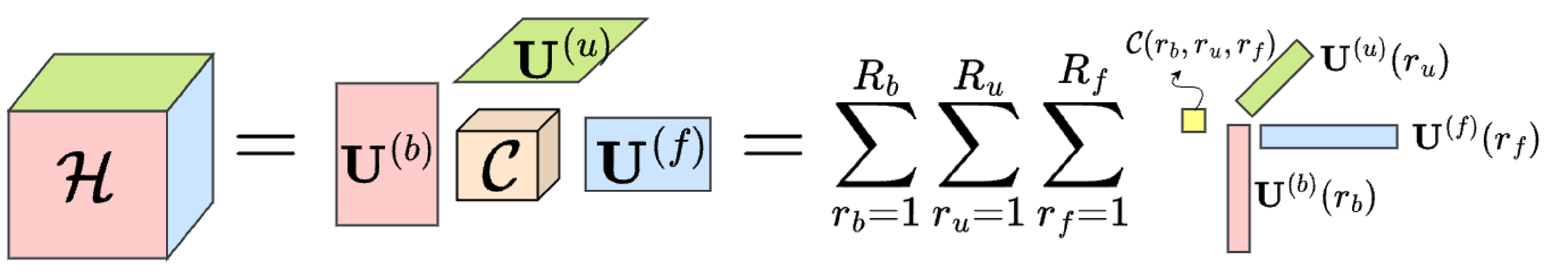

Figure 3. Tucker decomposition of channel tensor

Where $\mathbf{U}^{(b)} \in \mathbb{C}^{R_b \times N_b}$, $\mathbf{U}^{(u)} \in \mathbb{C}^{R_u \times N_u}$ and $\mathbf{U}^{(f)} \in \mathbb{C}^{R_f \times N_f}$ are orthonormal matrices, and $\mathcal{C} \in \mathbb{C}^{R_b \times R_u \times R_f}$ is core tensor. Here R_b, R_u, R_f are ranks of corresponding unfolding matrices. This decomposition can be calculated with interlacing computation of High-order singular value decomposition (HOSVD).

SVD-precoder via Tucker decomposition

Let us derive the Tucker decomposition for each element of tensor $\mathcal{H}$

$$\mathcal{H}(b, u, f) = \sum_{r_b=1}^{R_b} \sum_{r_u=1}^{R_u} \sum_{r_f=1}^{R_f} \mathcal{C}(r_b, r_u, r_f) \cdot \mathbf{U}^{(b)}(r_b, b) \cdot \mathbf{U}^{(u)}(r_u, u) \cdot \mathbf{U}^{(f)}(r_f, f) \tag{4}$$

where $b \in \{0, ..., N_b\}, u \in \{0, ..., N_u\}, f = \{0, ..., N_f\}$ Since we are interested in obtaining precoder for each subcarrier we can fix index f in previous formula to have:

$$\mathbf{H}_f = \sum_{r_b=1}^{R_b} \sum_{r_u=1}^{R_u} \sum_{r_f=1}^{R_f} \mathcal{C}(r_b, r_u, r_f) \cdot \mathbf{U}^{(f)}(r_f, f) \cdot \mathbf{U}^{(b)}(r_b) \circ \mathbf{U}^{(u)}(r_u) \tag{5}$$

Here $\circ$ denotes outer-product. Since only $\mathcal{C}$ and $\mathbf{U}^{(f)}$ depend on r_f we can replace them:

$$\mathbf{A}_f = \sum_{r_f=1}^{R_f} \mathcal{C}(r_b, r_u, r_f) \cdot \mathbf{U}^{(f)}(r_f, f) \tag{6}$$

Here $\mathbf{A}_f \in \mathbb{C}^{R_b \times R_u}$. Using SVD of $\mathbf{A}_f$ we get $\mathbf{A}_f = \tilde{\mathbf{U}}_f \tilde{\mathbf{\Sigma}}_f \tilde{\mathbf{V}}_f^H$

Finally we can rewrite equation 5 as following avoiding sum operators:

$$\mathbf{H}_f = \mathbf{U}^{(b)^T} \tilde{\mathbf{U}}_f \tilde{\mathbf{\Sigma}}_f \tilde{\mathbf{V}}_f^H \mathbf{U}^{(u)} \tag{7}$$

Since both $\mathbf{U}^{(b)}$ and $\tilde{\mathbf{U}}_f$ are , their product is also orthonormal, then

$$\mathbf{P}_f = \mathbf{U}^{(b)^T} \tilde{\mathbf{U}}_f \tag{8}$$

$\tilde{\mathbf{U}}_f = [\tilde{\mathbf{u}}_{f,r}]$, where $\tilde{\mathbf{u}}_{f,r} \in \mathbb{C}^{R_b \times 1}$,and $r \in \{0, ..., R\}$ If we are interested in specific singular vector $r \in \{0, ..., R\}$ it can be expressed as: $\mathbf{p}_{f,r} = \mathbf{U}^{(b)^T} \tilde{\mathbf{u}}_{f,r}$

It should be noted that results are equivalent up to a constant phase since SVD is invariant to the multiplication of each singular vector to $e^{i\phi}$, but this does not affect precoder performance.

Tucker decomposition spectrum analysis

Since Tucker decomposition can be represented as the sum of 1-rank tensors, we can analyze these components in terms of spatial power spectral density (SPSD) which describes the Angles of arrival/departure for each path and its delays. Let $\mathcal{A}$ be a function of SPSD $\mathbf{x} = \mathcal{A}(\mathcal{H})$, $\mathbf{x} \in \mathbb{R}^{K \times 1}$, where K is number of spectral components.

Then for any different SPSDs $\mathbf{x}$ and $\hat{\mathbf{x}}$ we can use cosine similarity function:

$$\alpha(\mathbf{x}, \hat{\mathbf{x}}) = \frac{\mathbf{x}^T \hat{\mathbf{x}}}{\|\mathbf{x}\|_2 \|\hat{\mathbf{x}}\|_2} \tag{9}$$

Proposed scheme was tested with Sionna Raytracing channel measurements in different scenarios: Line-of-sight (LOS), Non-line-of-sight (NLOS) and NLOS with dominating diffraction (NLOS DIFF)

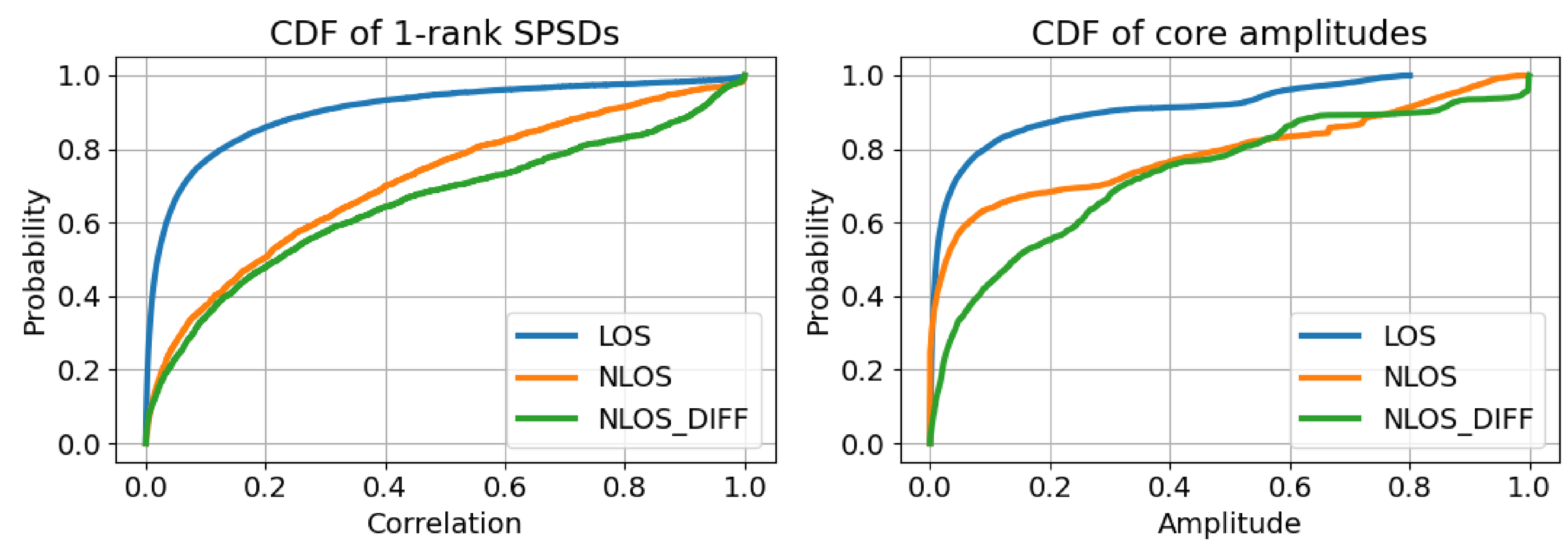

Figure 4. Simulation results. Left: Cumulative distribution function (CDF) of 1-rank tensor cross-correlations; Right: CDF of absolute value of core tensor elements

Parameter type	Parameter value
Channel model type	Sionna raytracing
BS/UE antenna configuration	16×8×2 / 2×1×2
Subcarrier spacing	360 kHz
Bandwidth	184 MHz
Number of subcarriers	512
Central frequency	6 GHz

Table 1. Simulation configuration

Conclusions

- Tucker decomposition can be used for calculating the same SVD-precoder as independently calculated on each subcarrier. This fact is important for other wireless channel applications such as distributed joint precoder calculation.
- Different 1-rank tensor components obtained from Tucker decomposition can describe different propagation paths, also described paths tend to overlap especially in NLOS scenario.
- Core tensor is sparse for all scenarios, which means that wireless channel data could be efficiently compressed using tensor decompositions.
- Generated by Sionna LOS channels contains many paths obtained from multiple reflections. However in presence of noise most of such paths won't be available.