Usage of Tensor Methods in Problems of Searching for Transport Equilibrium: an Overview

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ПЕРВАЯ МЕЖДУНАРОДНАЯ ШКОЛА-КОНФЕРЕНЦИЯ ПО ТЕНЗОРНЫМ МЕТОДАМ В МАТЕМАТИКЕ И ЗАДАЧАХ ИСКУССТВЕННОГО ИНТЕЛЛЕКТА

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MOTIVATION

With the growth of cities and road networks, the problem of constructing transport models has become particularly relevant. Globally, models can be divided into 2 types: ***static*** and ***dynamic*** models.

Static models are *less sensitive to input data* because they do not take into account how traffic conditions change over time. The *basis of such models is link performance functions*. However, the graph of the city's transport network is quite large, especially when modelling urban agglomerations with a large number of correspondence from the suburbs.

That is why some assumptions are introduced into the model, for example, about the independence of different types of transport. This makes the model less descriptive of the real situation.

Dynamic models take into account changes in the traffic situation over time, but *require more input data* for this. Finding balance becomes even more difficult. So, it is of interest to search for certain patterns in the behaviour of passengers, their departure times, travel purposes and the source and destination points of the traffic. For example, our earlier work proposed a framework for static models that allows the model to take into account the split into different types of transport, types of passengers and layers of demand. It is interesting to consider the possibility of identifying clusters using a tensor representation of data and their decomposition in order to use the obtained information to increase the efficiency and descriptive ability of both static and dynamic models.

In the picture there is a schema of classical 4-stages transport model. [1]

TENSOR DECOMPOSITION

Canonical polyadic decomposition. It is also called as parallel factor analysis, canonical decomposition, and topographic components model. The main idea of CP decomposition is to factorize a tensor as the sum of a series of rank-one tensors.

$$\mathcal{A} \approx [\lambda; A^{(1)}; A^{(2)}; \ldots; A^{(N)}] \equiv \sum_{r=1}^{R} \lambda_r a_r^{(1)} \circ a_r^{(2)} \circ \cdots \circ a_r^{(N)},$$

where $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$, $\lambda \in \mathbb{R}^R$, $A^{(n)} \in \mathbb{R}^{I_n \times R}$.

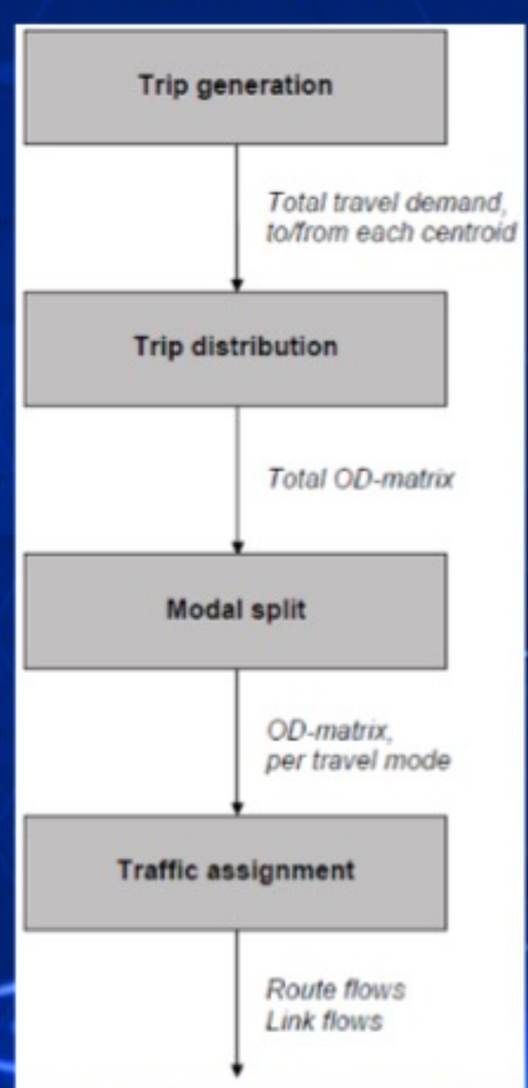
Tucker decomposition. It is called as N-mode PCA, N-mode SVD, and three-mode factor analysis in previous studies. the decomposition of a tensor by Tucker decomposition is similar to the decomposition of a matrix by PCA. Take a three-order tensor for example, it decomposes the tensor into a core tensor multiplied by a matrix along each mode.

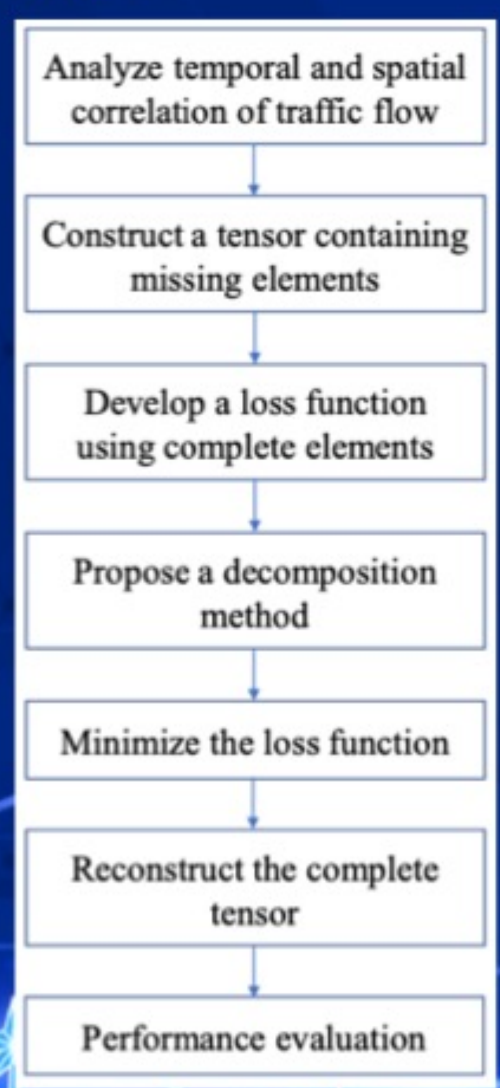
$$\mathcal{A} \approx C \times_1 A^{(1)} \times_2 A^{(2)} \ldots \times_N A^{(N)}$$

APPLICATION IN TRANSPORT MODELS

Traffic data imputation.

Although a large number of advanced sensors have been installed on highways, missing data problem is still unavoidable in traffic dataset because of the equipment failure, communication outage, and electricity interruption. The existence of missing data brings enormous challenges for all these different transportation applications.





From earlier studies, it can be found PCA-based imputation method could consider the daily, weekly, monthly, and spatial correlations of traffic data and construct traffic data into different matrix form to capture these correlations. By utilizing the multiple correlations of traffic data, these methods often outperform traditional time series and interpolation methods.

In higher order data it is known that CP decomposition was used to solve real-life problems with imputations in data. For example, it was used to impute public transport smart card transactions in metro stations in Hong Kong [2] and to impute OD matrix data from Urban rail transit including traffic passenger volumes in Xian [3]

Traffic state prediction

Traffic state prediction aims to estimate traffic flow, travel speed, or travel time of a road segment within a future time window which can contribute to travel route planning and navigation, dynamic traffic control, and other applications. Tensor decomposition-based imputation method has been proved that have high-performance for processing incomplete traffic data. The task of traffic state prediction can transfer to fill the missing entries locating in the end of time series.

Analysis of travel pattern

Origin-destination (OD) data plays an important role in fundamental traffic engineering filed such as transport planning, operations and management to promote the efficiency of urban transportation system [4]. The applications of tensor decomposition for processing OD data mainly including prediction, denosing and imputation by the low rank tensor approximation algorithms. Moreover, tensor decomposition has been a primary method for discovering the spatial and temporal patterns of human mobility and understanding the urban regional dynamic from the record of public transport smart card, vehicle GPS data and traffic flow data. [5]

TRAFFIC ASSIGNMENT PROBLEM (TAP)

At the same time, finding balance is still quite a difficult task. In this regard, the idea of using high-order optimization methods, tensor methods and their comparison with optimization methods that are standardly used to find equilibrium in transport (universal gradient method, universal method of similar triangles) arises.

Optimization problem formulation of TAP can be also shown in form of variational inequality.

$$\mathbf{c}(\hat{\mathbf{h}}) \cdot (\hat{\mathbf{h}} - \mathbf{h}) \leq 0 \qquad \forall \mathbf{h} \in H$$

It was suggested by Nesterov [6] to use tensor methods to solve VI problems. Proposed methods are based on a concept of Reduced Gradients. It also was shown that it benefits from hot-start which can be useful in TAP.

CONCLUSION

However, it was shown even in practice that tensor decomposition methods can be useful for transport modelling, there is still an issue with curse of dimensionality due to specific of data. The same holds for TAP methods. So all these techniques should be compared carefully with existing methods and may be modified somehow to work with real-life transportation graphs.

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