Fourier Neural Networks and Kolmogorov-Arnold Neural Networks for MRI Reconstruction

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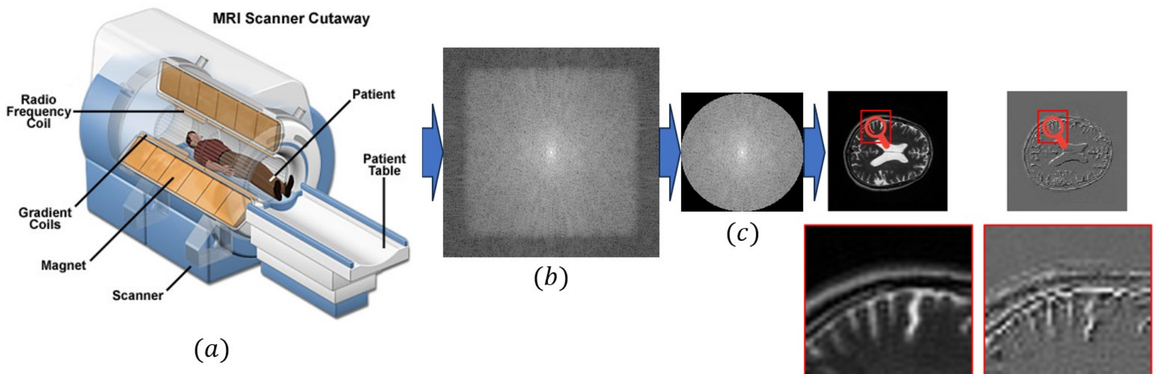
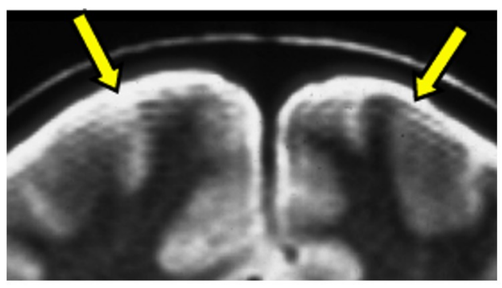
Introduction

- Magnetic Resonance Imaging (MRI) is a non-invasive imaging technology that produces three-dimensional detailed anatomical images.
  As with any other imaging modality, MRI is vulnerable to artifacts.
- Gibbs ringing is a common type of distortion specific for MR images.

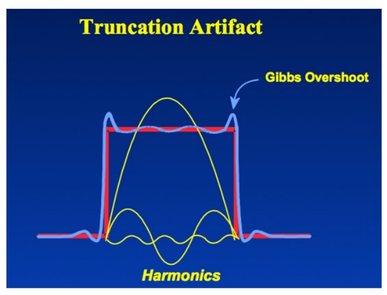


Low-resolution magnetic resonance image data acquisition.

(a) – MRI scanner;
(b) – Fourier spectrum, acquired by (a);
(c) – Fourier spectrum truncation (hardware limitation);
(d) – inverse Fourier transform of (c);
(e) – residual image: (d) − (a), demonstrating the distortion.



Oscillations may be observed around the edges of axial brain images due to the signal intensity difference at locations, such as the CSF-spinal cord or the skull-brain interface.



Ringing occurs, as the Fourier series cannot represent a discontinuity with a finite number of harmonics.

Goals

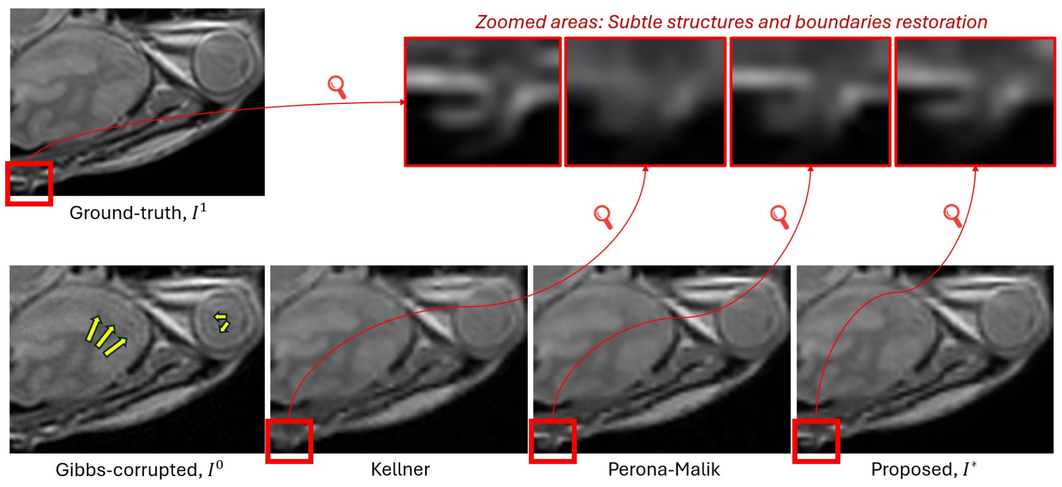
MR images enhancement

DL algorithm interpretability & practicality

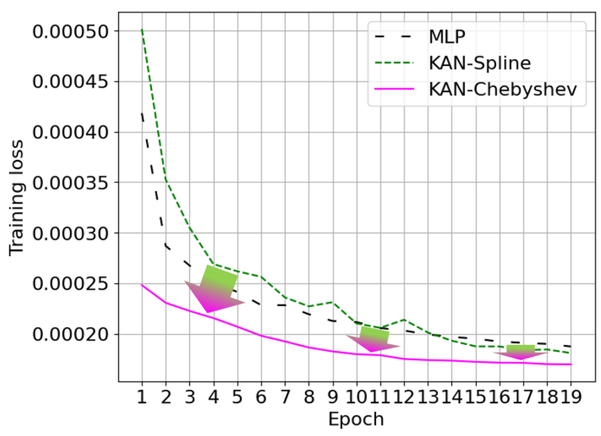
- Suppress Gibbs ringing and noise.
- Preserve essential image features (e.g. edges).
- Use physics-guided DL architectures.
- Make algorithm lightweight and GPU friendly out-of-box.

Results

1. *Regularization effect* of FNO and KAN, as DL approximators over the classical deringing algorithms: Perona-Malik and Kellner.



2. *Accelerated convergence* of CNN, packed with KAN deep feature extractor.



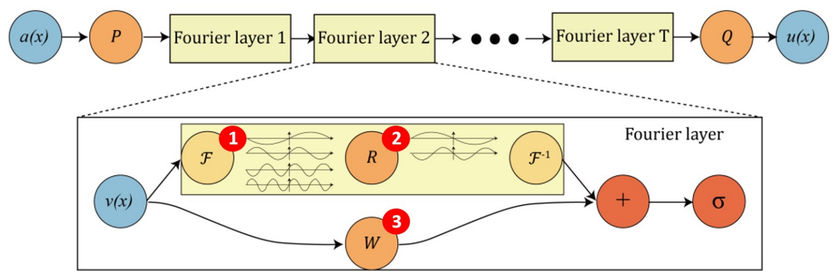
3. *Fast inference* on GPU.

Table: Inference time.
(Python realizations)

Method		Inference time, ms	
		CPU	GPU
Classical	Kellner	900	
	Perona-Malik	1	
Deep Learning	FNO	160	50
	MLP	8	4
	KAN-Spline	93	27
	KAN-Chebyshev	36	6

Method

1. Fourier Neural Operator



The FNO architecture.

— Lift to a higher dimension channel space by a neural network *P*.
— Apply *T* layers of integral operators and activation functions.
— Project back to the target dimension by a neural network *Q*.

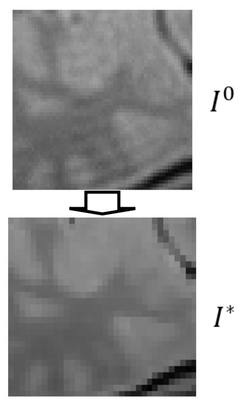
- FNO is motivated by the process of solving PDE via the Green's function:

Given PDE:
$(\mathcal{L}_a u)(x) = f(x)$

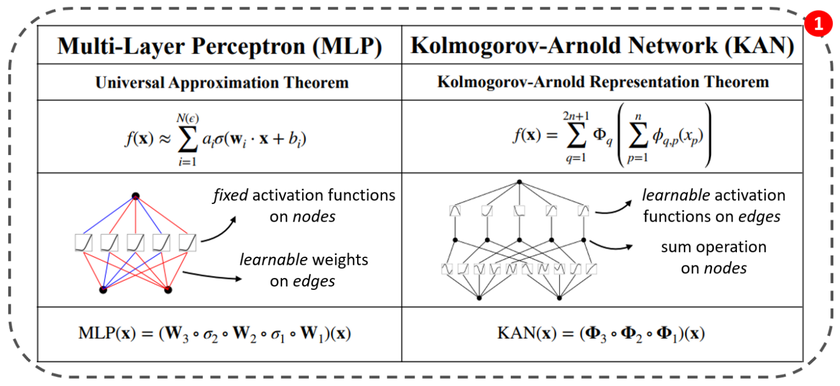
Analytical solution:
$u^*(x) = \int_\Omega G_a(x,y) f(y) dy$

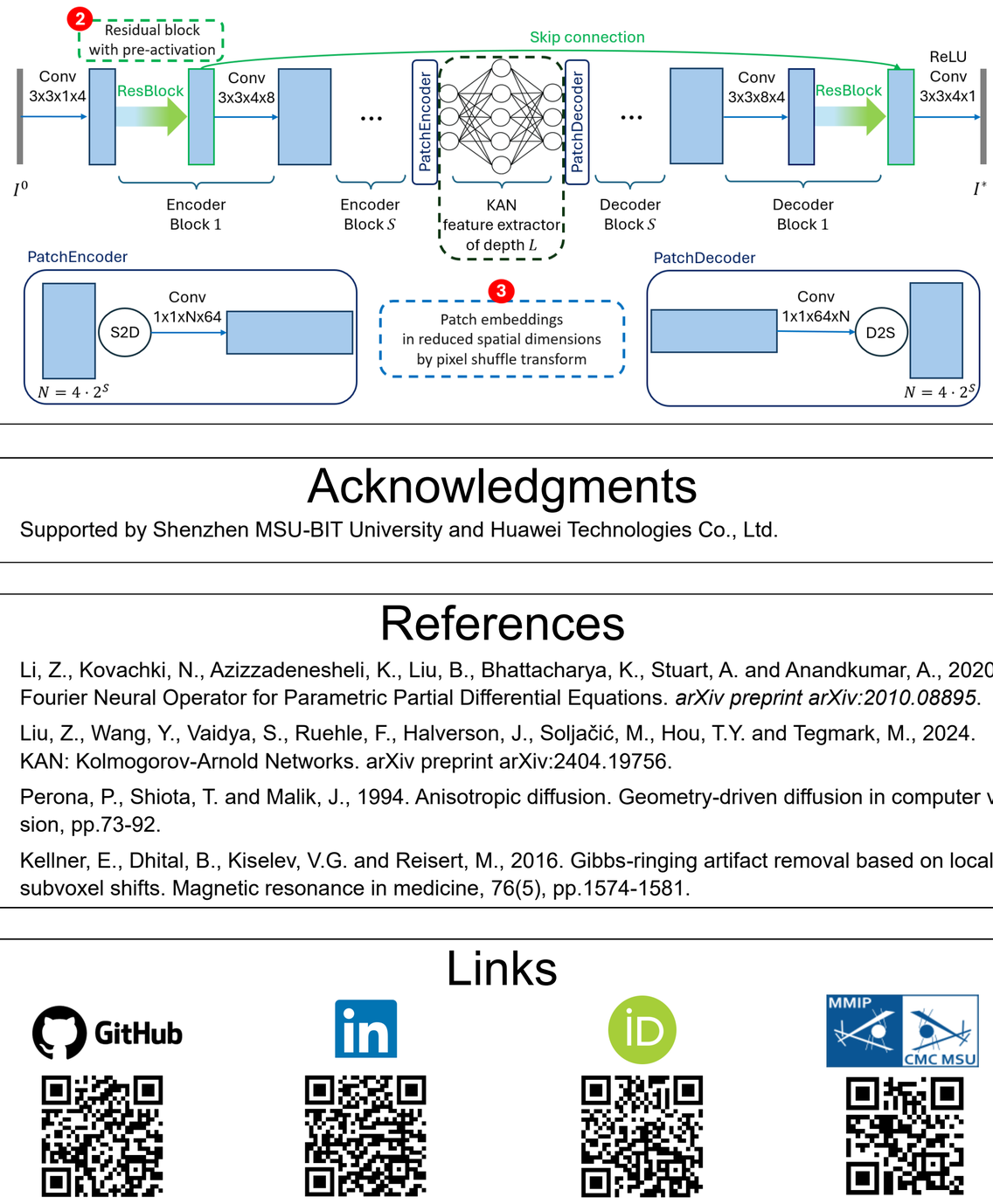
Numerical iterative architecture:
$v_{t+1}(x) = \sigma\left(W v_t(x) + \int_D \kappa_\phi(x, y, a(x), a(y)) v_t(y)\, \nu_x(dy)\right)$

For example, Perona-Malik PDE:
$\begin{cases} \frac{\partial I}{\partial t} = div(c(x,y,t) \cdot \nabla I) \\ I(x,y)|_{t=0} = I^0(x,y) \end{cases}$



2. Kolmogorov-Arnold Network





Acknowledgments

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References

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Links

GitHub

CMC MSU