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Mosaic-skeleton format in method of moments for large-scale scattering problems

Mass Ilya ♠◇

♠ Lomonosov Moscow State University ◇ Marchuk Institute of Numerical Mathematics

†massia@my.msu.ru

Electrodynamics problem

We consider the three-dimensional scattering problem of a monochromatic electromagnetic wave by a system of perfectly conducting surfaces (closed or open) Σ.

$$\operatorname{rot}\mathbf{E} = \mathbf{i}\omega\mu\mathbf{H}, \qquad \operatorname{rot}\mathbf{H} = -\mathbf{i}\omega\varepsilon\mathbf{E},$$

We also assume that that radiation sources producing the incident field, which can have the form of a plane wave

$$\boldsymbol{E}_{inc} = \boldsymbol{p}e^{\mathbf{i}k(\boldsymbol{v},\boldsymbol{x})}, \quad \boldsymbol{H}_{inc} = \frac{\boldsymbol{v}\times\boldsymbol{E}_{inc}}{\mathbf{i}\omega\mu}, \tag{1}$$

here $\boldsymbol{v}$ – unit vector of the incident wave direction, $\mathbf{p}$ – unit vector of the wave polarization, $\mathbf{v}\perp\mathbf{p}$, k is the wave number defined by the formula

$$k = \omega\sqrt{\varepsilon_0\mu_0}$$

On the surface of an ideal conductor the condition of absence of the tangent component of the total electric field is fulfilled:

$$\mathbf{n}(x)\times\mathbf{E}(x) = 0, \quad x\in\Sigma. \tag{2}$$

Integral equation

According to [1] an electric field is sought in the form

$$\mathbf{E} = \mathbf{E}_{inc} + \frac{\mathbf{i}}{\omega\varepsilon_0}\boldsymbol{\mathcal{K}}[\Sigma,\mathbf{j}],$$

where $\boldsymbol{\mathcal{K}}$ denotes the operator

$$\boldsymbol{\mathcal{K}}[\Sigma,\mathbf{g}](x) = \operatorname{rot}\operatorname{rot}\int_\Sigma \mathbf{g}(y)F(x-y)d\sigma_y, \quad F(x-y) = \frac{e^{\mathbf{i}k|x-y|}}{4\pi|x-y|}.$$

We will search for unknown vector fields $\mathbf{j}$ – tangential vector field on the surface of Σ, then the boundary condition can be rewritten as a surface integral integral equation:

$$\mathbf{n}\times\left(\mathbf{E}_{inc} + \frac{i}{\omega\varepsilon_0}\boldsymbol{\mathcal{K}}[\Sigma,\mathbf{j}]\right) = 0, \quad x\in\Sigma. \tag{3}$$

An important characteristic of the scattered electromagnetic field in the far field is the radar cross section (RCS). With known currents $\mathbf{j}$, the RCS can be calculated by the formula

$$\sigma(\boldsymbol{\tau}) = \frac{4\pi}{|\mathbf{E}_0|^2}\left|\int_\Sigma e^{-\mathbf{i}k(\boldsymbol{\tau},y)}\frac{\mathbf{i}k^2}{\omega\varepsilon_0}\left(\boldsymbol{j}-\boldsymbol{\tau}(\boldsymbol{j},\boldsymbol{\tau})\right)dy\right|. \tag{4}$$

Numerical method

The surface of an ideal conductor is approximated by a system of triangular cells σ_i, the approximation of the unknown currents $\mathbf{j}$ is sought as a linear combination of RWG basis functions [2]:

$$\mathbf{j}(x) = \sum_{i=1,N_v} j_i\mathbf{v}_i(x). \tag{5}$$

Here $\mathbf{v}_i(x)$ is a basis function bound to an internal edge that is common to some two cells.

The system of integral equations (3) is solved numerically by the Bubnov-Galerkin method using the basis functions presented above. As (3) is hypersingular integral equation, for calculating matrix elements an equivalent formula with weak singularity is used:

$$(\mathbf{v}_i,\boldsymbol{\mathcal{K}}[\Sigma,\mathbf{v}_j])_\Sigma = \int_{\sigma_i^{1,2}}\int_{\sigma_j^{1,2}}\left(k^2\mathbf{v}_i(x)\cdot\mathbf{v}_j(y) - D_iD_j\right)F(x-y)d\sigma_y d\sigma_x \tag{6}$$

Weak singularity in (6) calculated analytically (see [1]), remaining part of formula evaluated numerically with adaptive numerical integration.

With dense matrix of linear equation, mosaic-skeleton approximation and GMRES iterative solver with multiple right sides is used. Thus, numerical method has such approximations as:

- Mesh with diameter d as approximation of initial geometry with
- Adaptive numerical integration as matrix elements approximation
- Mosaic skeleton format as approximation of whole matrix
- GMRES as iterative solver with linear system.

Experiments

Numerical experiments were performed on metal cylinder with r=7.5cm and h=25cm, backscattering RCS is considered. Thus, linear system were solved for multiple right sides (360 and 720 for 2-4 and 4-8 GHZ respectively). Both horizontal and vertical polarization were modeled with the similar time, compression and error/residual results.

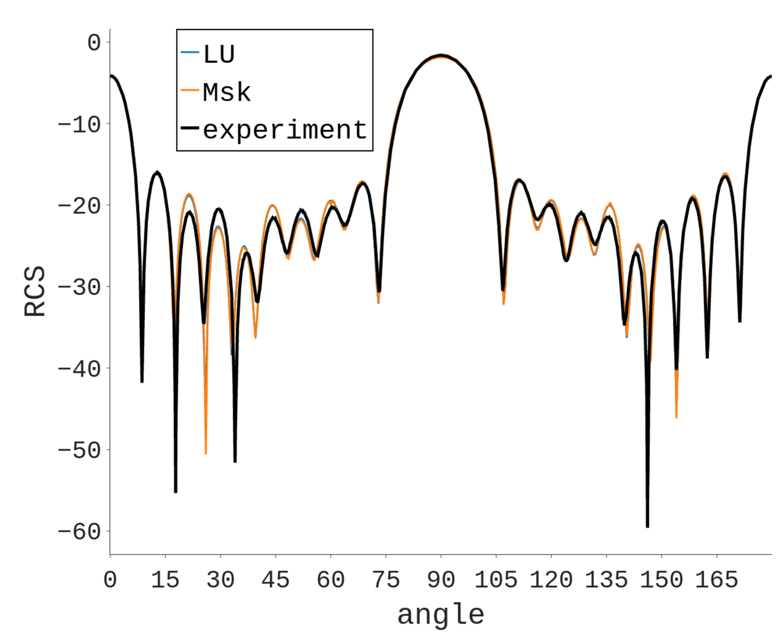
W, Ghz	size	kd	full matrix, s	LU, s	compress	approx, s	GMRES, s	RCS rtol
2	21k	0.3	201	16	10%	83	94	4e-3
4	40k	0.43	975	118	5.8%	222	581	1e-2
8	85k	0.6	4133	1025	3.7%	580	3980	1e-2
16	85k	1.25	4322	1000	5%	754	11000	0.17

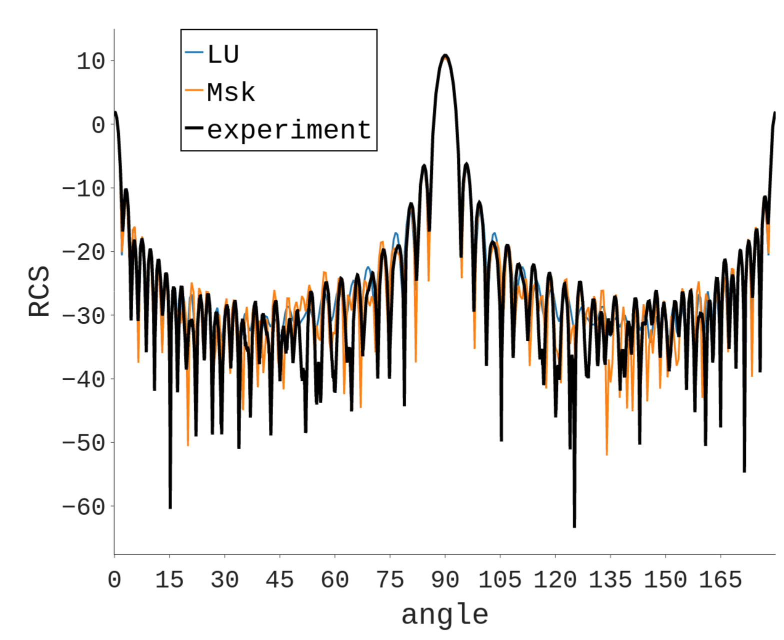
Table 1: comparison of MSk+GMRES and LU full matrix solver

w	kd	rtol	compress	time
4	0.43	1e-2	4.3%	193s
		1e-3	5.8%	222s
		1e-4	7.6%	260s
		1e-5	10%	625s

w	kd	rtol	compress	time
8	0.87	1e-2	5%	226s
		1e-3	6.8%	265s
		1e-4	9%	313s
		1e-5	11.8%	557s

Table 2: MSk compression on fixed mesh





References

[1] W.C. Gibson. *The Method of Moments in Electromagnetics*. CRC Press, 2021.

[2] Sadasiva Rao, Donald Wilton, and Allen Glisson. Electromagnetic scattering by surfaces of arbitrary shape. *IEEE Transactions on antennas and propagation*, 30(3):409–418, 1982.