Taking into account the low-rank structure of the matrix of perturbation coefficients in the problem of compensation of nonlinear distortions in fiber optics

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1. Introduction

Fiber-optic systems form the backbone of modern telecommunication networks. However, nonlinear signal distortion in optical fibers can significantly impact transmission quality in fiber optic communication lines. Therefore, there is a need to develop signal processing methods that can improve transmission quality. Nonlinear distortion caused by the medium is one of the main limiting factors for signal transmission over fiber-optic networks.

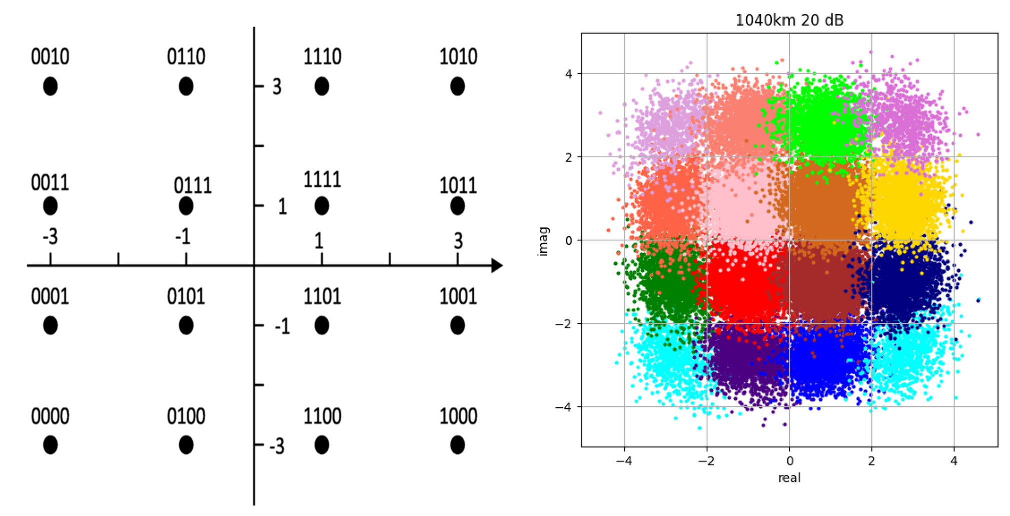

Figure 1: (left) Input complex amplitudes. (right) Output complex amplitudes.

Compensating for nonlinear distortions in an optical fiber presents an inverse problem. The goal is to reconstruct the complex amplitudes that were input into the waveguide from the distorted output signal.

2. Perturbation-based model

The propagation of an optical signal in a fiber is mathematically described by the nonlinear Schrödinger equation (NLSE)

$$\frac{\partial}{\partial z}u(t,z) + i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u(t,z) + \frac{\alpha}{2}u(t,z) = i\gamma|u(t,z)|^2 u(t,z).$$

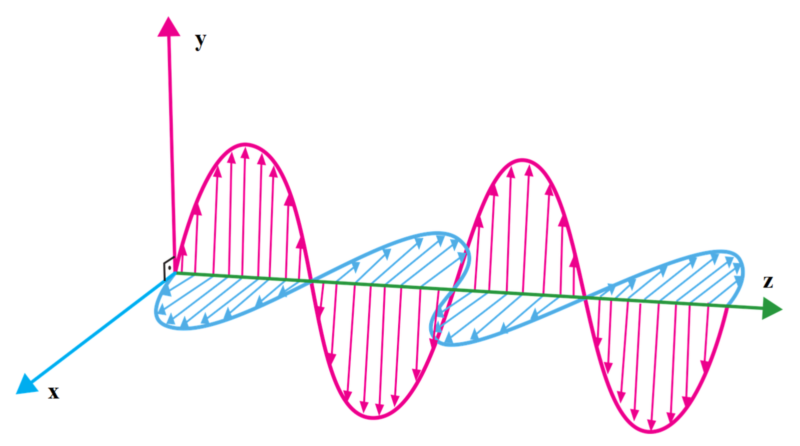

Figure 2: Illustration of a polarized signal

To study this phenomenon, a quasi-linear model of signal propagation is considered, which sees nonlinearities as small perturbations to the linear solution. The analysis of first-order perturbations allows us to transition from the nonlinear Schrödinger equation (NLSE) to the following model

$$y(k) = b \cdot x(k) + \Delta x(k) + \eta,$$

where $x(k)$ represents the input complex amplitudes in the k-th time frame, and $y(k)$ represents the output complex amplitudes in the k-th time frame. $\Delta x(k)$ represents the estimation of nonlinear distortions accumulated during signal propagation through a fiber, caused by the cubic nonlinearity of the medium and considering the interaction between triplets of neighboring signals over time. This estimation expressed by the following formula

$$\Delta x(k) = \sum_{i=0}^{1}\sum_{n=-N}^{N}\sum_{m=-M}^{M} C^i_{m,n} x^0(k+m) x^i(k+n) x^{i*}(k+m+n),$$

here i stands for polarization and $\cdot^*$ means complex conjugation. By employing the least squares method to determine the perturbation coefficients $C^i_{m,n}$, the strategy effectively reduces the computational complexity. This approach is highly efficient, as it simplifies the process compared to calculating coefficients through multidimensional integrals. Ultimately, by minimizing the number of various coefficients, the primary goal is to decrease the overall computational complexity of the model.

3. Complexity reduction

Considering the coefficient matrices $C^i \in \mathbb{R}^{M+1\times N+1}$ in the form of a skeleton decomposition

$$C^i = \sum_{r=1}^{R} u^i_r {v^i_r}^T.$$

This factorized representation allows us to search for perturbation coefficients in a low-rank form using the alternating least squares (ALS) procedure. It alternates between fixing U and solving for V, then vice versa, using the least squares criterion until convergence.

To reduce the number of different coefficients, consider the Haar transformation matrix

$$H = \left\{\frac{1}{\sqrt{2n}} h_k\left(\frac{j}{2n}\right)\right\}_{k,j=0}^{2n-1}, \quad h_k(x) = \begin{cases} 2^{\frac{k}{2}}, & x \in \Delta^{k+1}_{2m} \\ -2^{\frac{k}{2}}, & x \in \Delta^{k+1}_{2m+1} \\ 0, & \text{else} \end{cases}.$$

The Haar transformation matrix, being an orthogonal matrix, is particularly effective in transforming data into a simpler form. It can be used to represent coefficients more compactly, thus reducing the computational complexity. In our case, we apply the Haar transform to the vectors u^i_r and v^i_r

$$\Delta x(k) = \sum_{i=0}^{1}\sum_{r=1}^{R} (H \cdot u^i_r)^T H A^i(k) H^T (H \cdot v^i_r).$$

To reduce the number of different perturbation coefficients, the small magnitude components of the vectors Hu^i_r and Hv^i_r are discarded, and the inverse Haar transform is applied. This transformation makes it possible to obtain an adaptive piecewise constant approximation of the vectors u^i_r, v^i_r.

Figure 3: The absolute values of the components of the Hu^0_1 vector sorted in descending order for the model $M = N = 128$ and $r = 2$.

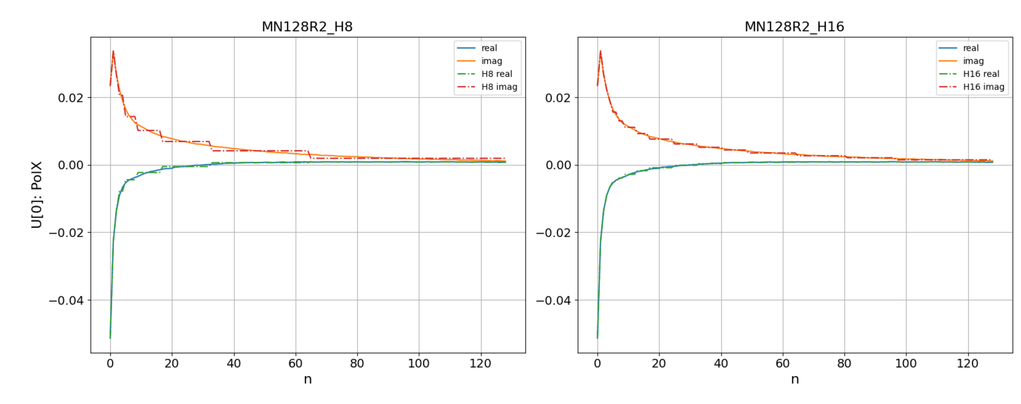

Figure 4: The reconstructed vector $\tilde{u}^0_1$ is obtained after discarding small coefficients in the image of Hu^0_1 with 8 and 16 nonzero components for the model with parameters $M = N = 128$ and $r = 2$.

Next, we consider the matrices from the reconstructed vectors $\tilde{C}^i = \sum_{r=1}^{R} u^i_r (v^T_r)^i$ as approximations to the original matrices C^i. By applying threshold filtering to the elements of the $\tilde{C}^i$ matrices, we obtain the resultant $\hat{C}^i$ matrices.

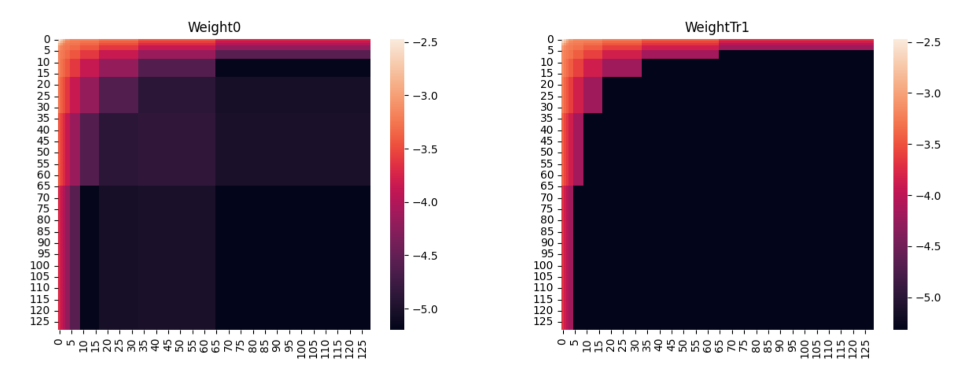

Figure 5: (left) The coefficient matrix $\tilde{C}^0$ was obtained from the vectors $\tilde{u}^0_r$ and $\tilde{v}^0_r$ using 8 steps. (right) The $\hat{C}^0$ matrix was filtered with a threshold of $\delta = 0.01$.

# steps	8	16	32	128	no model
NMSE, dB	-13.742	-13.851	-13.874	-13.877	-11.20

Table 1: The dependence of the normalized mean square error (NMSE) on the number of steps in $\tilde{u}^i_r$, $\tilde{v}^i_r$ for a model with parameters $N = M = 128, R = 2$.

(N, M)	R	N_{uc}	$\frac{th_{C0}}{\lvert max \rvert}$	$\frac{th_{C1}}{\lvert max \rvert}$	NMSE, dB	NMSE (full)
(128, 128)	4	45	0.08	0.09	-13.03	-14.44
(128, 128)	4	60	0.03	0.03	-13.87	
(128, 128)	4	66	0.01	0.01	-14.09	
(128, 128)	4	all	0	0	-14.16	

Table 2: The dependence of NMSE on the number of different coefficients in the matrices $\hat{C}^0$ and $\hat{C}^1$. The thresholds th_{C^0} and th_{C^1} are chosen so that the matrices have the same number of coefficients.

4. Conclusions

- The Haar transform reduces the computational complexity of the model.
- Due to piecewise constant approximation, it is possible to significantly reduce the number of different components in the vectors u^i_r and v^i_r, thereby reducing the number of multiplications required to calculate the nonlinear correction.
- Discarding small coefficients leads to minor losses in the quality of the restored signal.

References

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