Atmospheric Predictability: an adjoint perspective

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NCAR

Symposium in honor of the 90th anniversary of the birth of G.I. Marchuk

U.S. DEPARTMENT OF ENERGY
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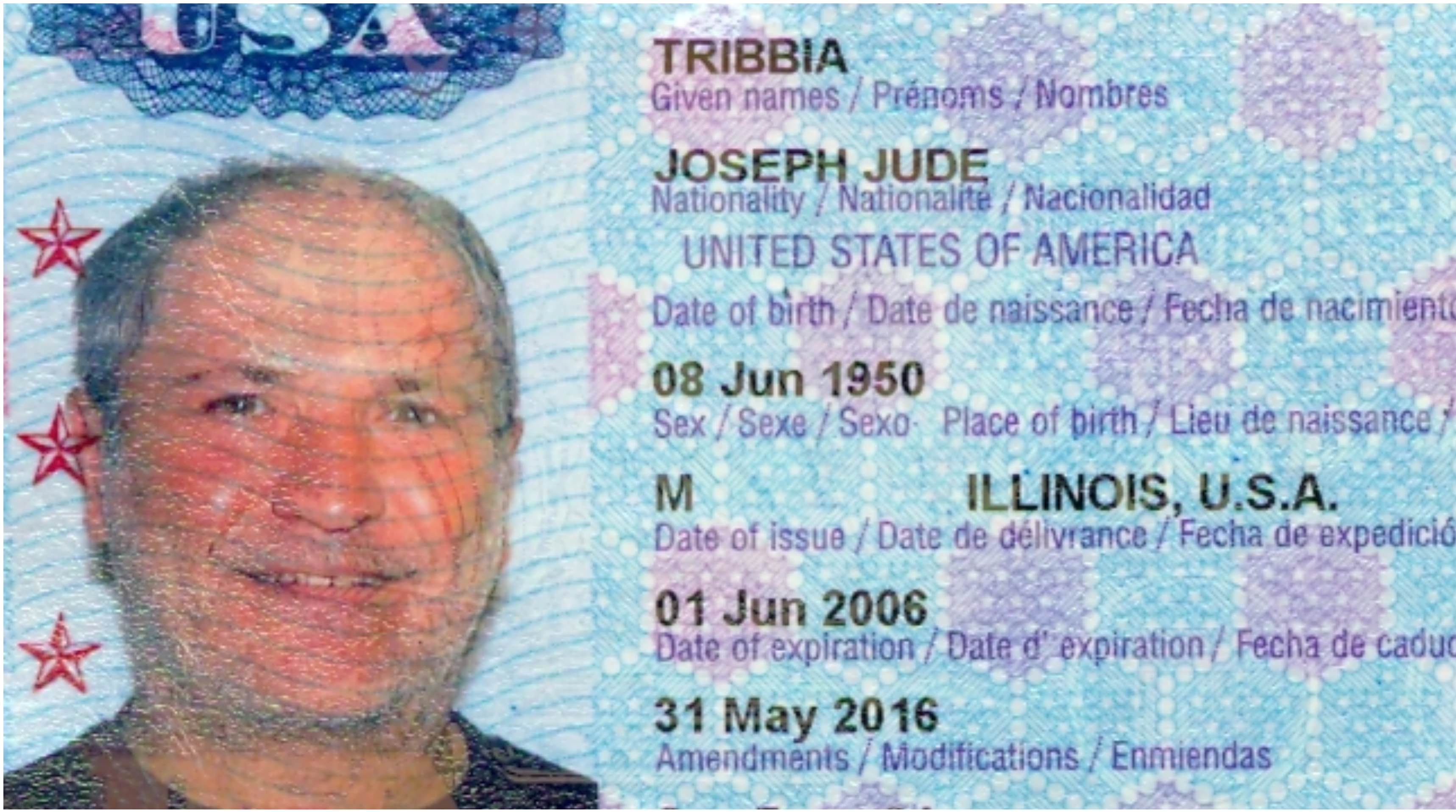
Remarkable Scientist

- **Gury Marchuk**
- **From Wikipedia, the free encyclopedia**
- Gury Marchuk Born 8 June 1925 Petro-Khersonets, Orenburg Governorate, USSR Died 24 March 2013 Nationality Russian Alma mater Leningrad State University Thesis (1957) **Gury Ivanovich Marchuk** (Russian: Гурий Иванович Марчук; 8 June 1925 – 24 March 2013) was a prominent Soviet and Russian scientist in the fields of computational mathematics, and physics of atmosphere.[1] Academician (since 1968); the President of the USSR Academy of Sciences in 1986–1991. Among his notable prizes are the USSR State Prize (1979), Demidov Prize (2004), Lomonosov Gold Medal (2004).

Remarkable Coincidence



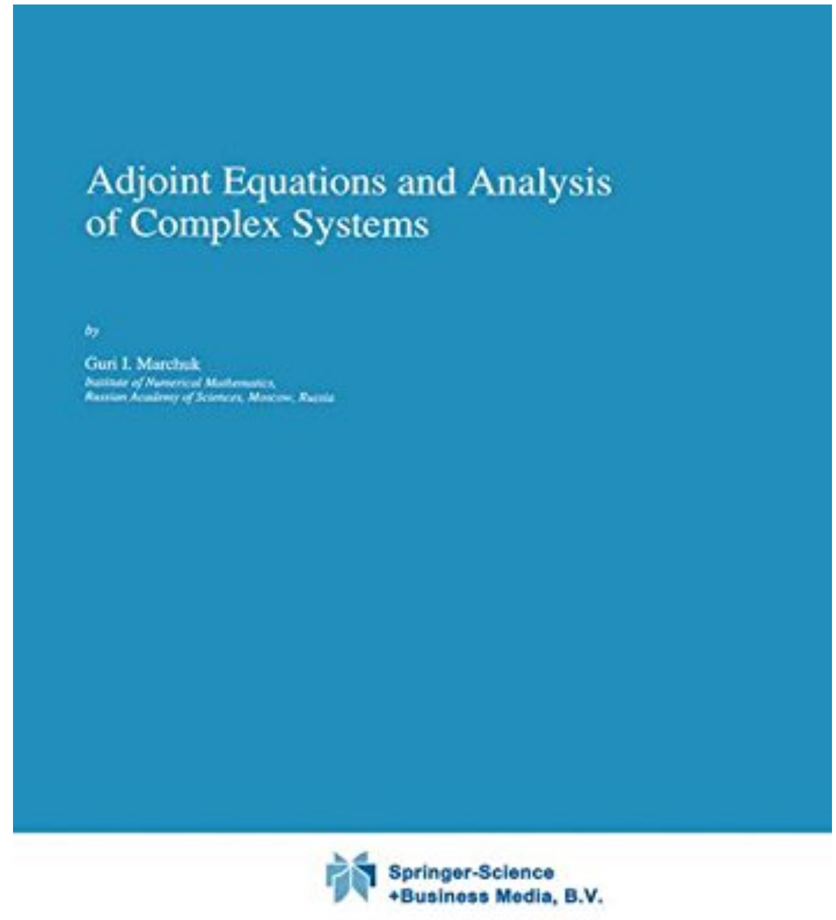
Outline

- Examine: Why/How do large scales lose predictability?

1) Adjoints + Predictability=Singular Vectors
2) Rational approach to ensemble
3) Fraternal twin experiments
4) Singular vectors vs cascade

Unaccounted Influence of Marchuk

- English versions of work in book appears in 1980's
- Dan Cacuci analyzes Climate Sensitivity using adjoint methods
- Use in Variational Data Assimilation
- Use in Generalized Stability and Ensemble Prediction



1. Adjoint Operators and Predictability

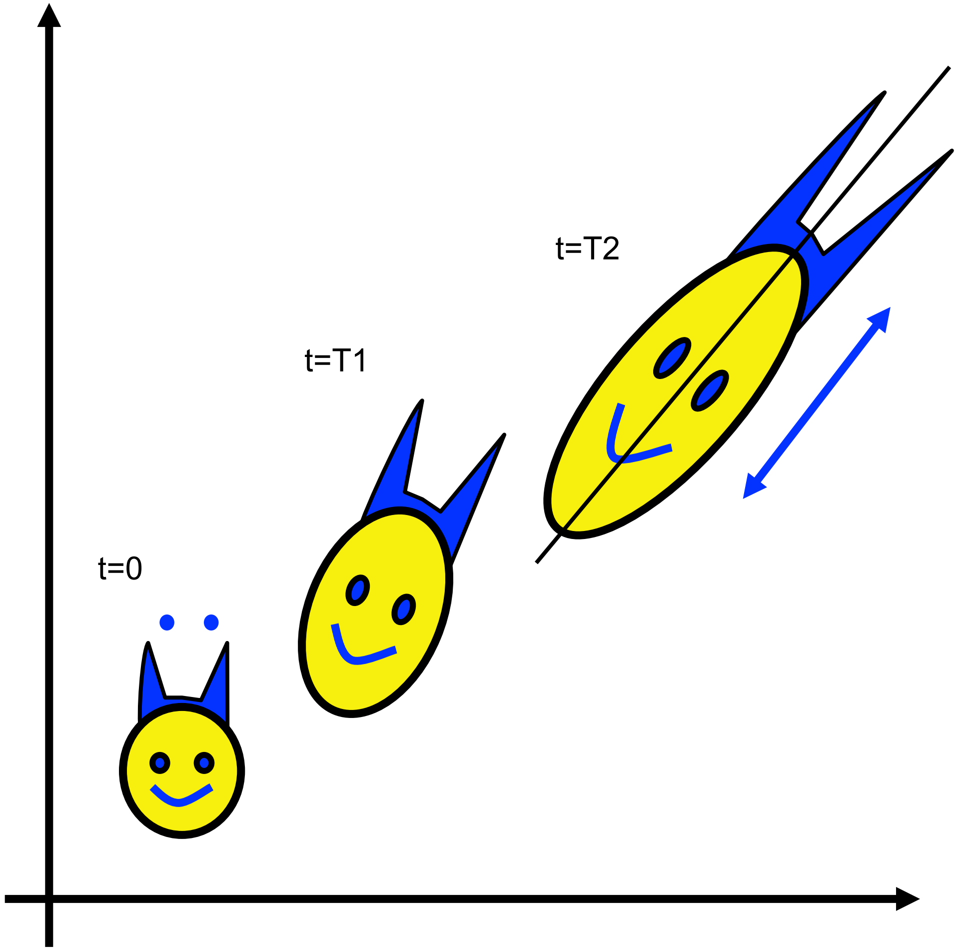
Clearly related

The ECMWF approach to the simulation of initial uncertainties

MOST DANGEROUS DIRECTIONS

Perturbations pointing along different axes in the phase-space of the system are characterized by different amplification rates. As a consequence, the initial PDF is stretched principally along directions of maximum growth.

The component of an initial perturbation pointing along a direction of maximum growth amplifies more than a component along another direction .



Singular vector definition: the linear equations

Consider an N-dimensional nonlinear system:

$$\frac{\partial y}{\partial t} = A(y,t)$$

Denote by *z′* a small perturbation around a time-evolving trajectory *z*:

$$\frac{\partial z'}{\partial t} = A_l(z,t)z' \qquad A_l(z,t) = \left.\frac{\partial A(z,t)}{\partial z}\right|_z$$

$$\frac{\partial z}{\partial t} = A(z,t)$$

The time evolution of the small perturbation *z′* is described to a good degree of approximation by the linearized system $A_l(z,t)$ defined by the trajectory.

Singular vector definition: the linear propagator

The perturbation *z'* at time *t* is given by the time integration from the initial state *z'(t=0)* of the linear system:

$$z'(t) = z'_0 + \int_0^t A_l(z,s)\,ds$$

The solution can be written in terms of the linear propagator *L(t,0)*:

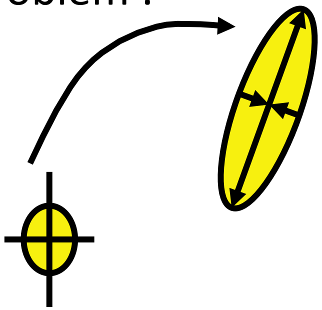
$$z'(t) = L(t,0)z'_0$$

The linear propagator is defined by the system equations and depends on the trajectory characteristics. The E-norm of the perturbation at time *t* is given by:

$$\left\| z'(t) \right\|^2 =< z'(t); Ez'(t) >=< L(t,0)z'_0; EL(t,0)z'_0 >$$

Simulation of initial uncertainties: the singular vector approach

The problem of the computation of the directions of maximum growth of a time evolving trajectory is solved by computing the singular vectors of $K=E^{1/2}LE_0^{-1/2}$, i.e. by Rayleigh-Ritz equivalent to solving the following eigenvalue problem :



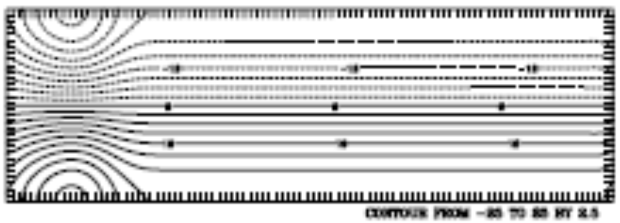
$$E_0^{-1/2}L^*ELE_0^{-1/2}v=\sigma^2 v$$

By definition, the singular vectors depend:

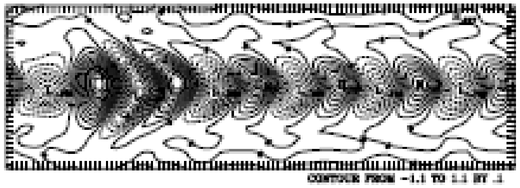
– on the **initial and final time metrics** $\boldsymbol{E_0}$ and $\boldsymbol{E}$;
– on the **linear propagator** ***L(t,0)***;
– on the **time-evolving trajectory** along which they are computed**;**
– on the **optimization time interval.**

In Ensemble Weather Prediction:
Singular vectors and Bred vectors are used

Basic state



S.V.



B.V.

Singular vectors are the fastest growing structures into the future
Bred vectors are the fastest growing structures from the past.

Prediction of a nascent (SV) or mature (BV) probability density
SVs are EOFs of unstructured initial errors
BVs are EOFs of structured initial errors
SVs evolve toward BVs

2. A Rational Approach to Ensemble Prediction

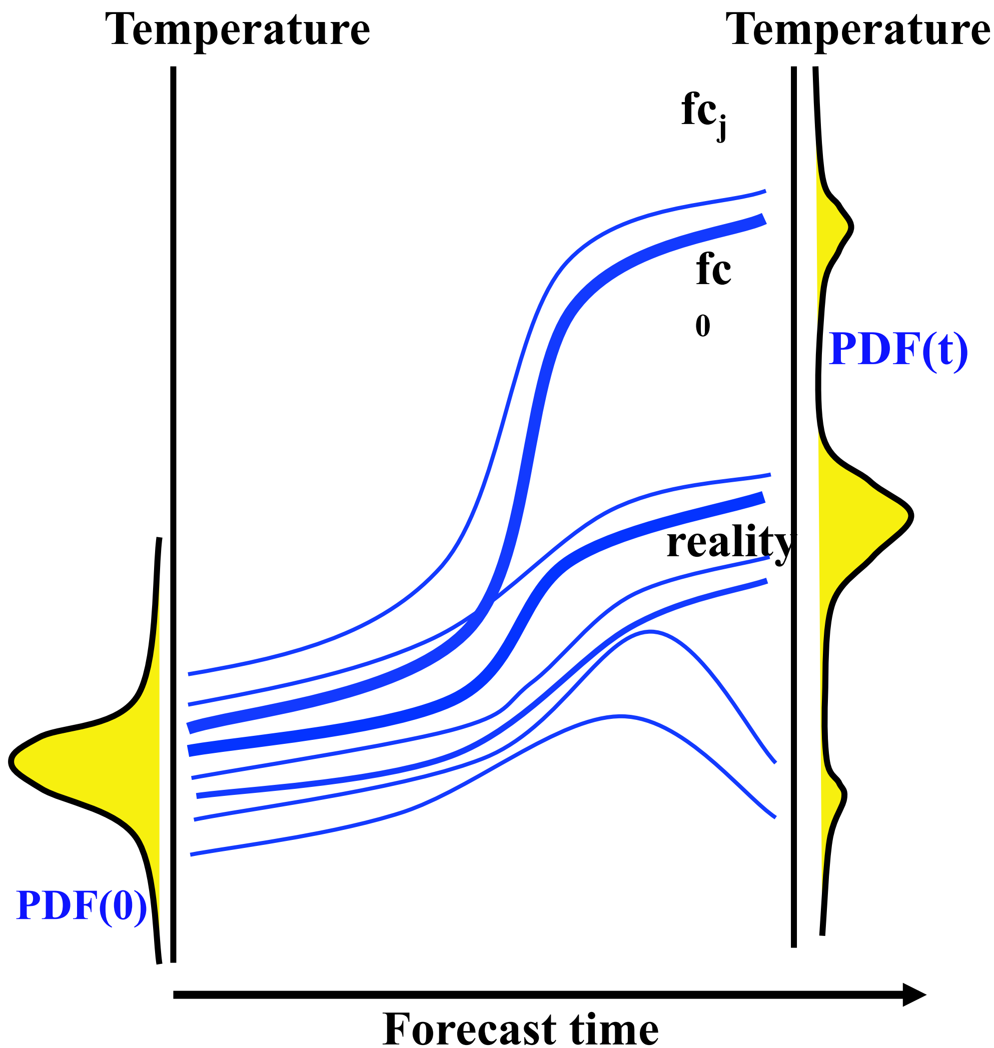
Not Just the 'Most Dangerous' Degrees of Freedom

The probabilistic approach to NWP: ensemble prediction

A complete description of the weather prediction problem can be stated in terms of the time evolution of an appropriate probability density function (PDF).

Ensemble prediction based on a finite small number of deterministic integrations appears to be the only feasible method to predict the PDF beyond the range of linear growth.

We must be strategic in sampling to capture the most important parts of PDF evolution



Probabilistic view of Predictability with a Hydrodynamic analogy

Given a linearized dynamical system

$$dx/dt = f(x,t) \approx Ax = V(x,t)$$

Consider the evolution of

a density of states $\quad p(x,t)$

Liouville equation $\quad \partial p/\partial t + \nabla \bullet (Vp) = 0$

Initially, p(x,0) is a tight distribution that dynamically broadens in time
For short time uncertainty is small and linearity of deviations is reasonable

For time order Δt Singular Vectors = Eigenvectors of the Rate of Strain Tensor for V
For a linear system SV's are EOF's of the PDF at $t=T_{init}$ and $t=T_{final}$

Operational Singular Vectors

500hPa Initial State
12 UTC 17 January 1987

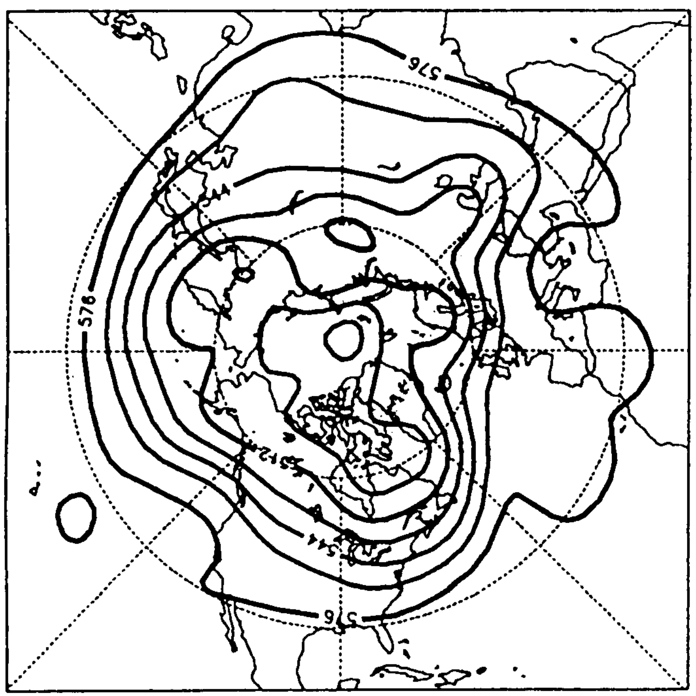


Fig. 1. 500 hPa geopotential height field at 12 UTC, 17.01.1987 (contour isolines every 160 m).

Six leading Singular Vectors
Note small scale and local nature

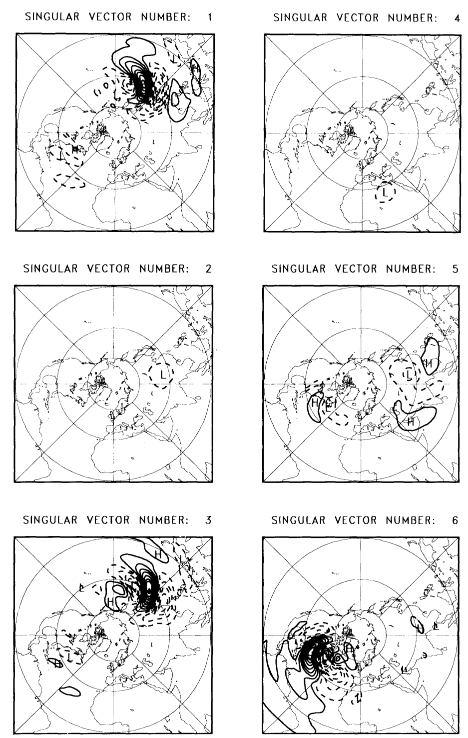


Fig. 3. Level-11 streamfunction of the first six SVs optimized over 24 h (increasing SV number from top-left t bottom-left, from top-right to bottom-right). The SVs are normalized to have unit total energy norm.

Sensitivity of growth rate to optimization time

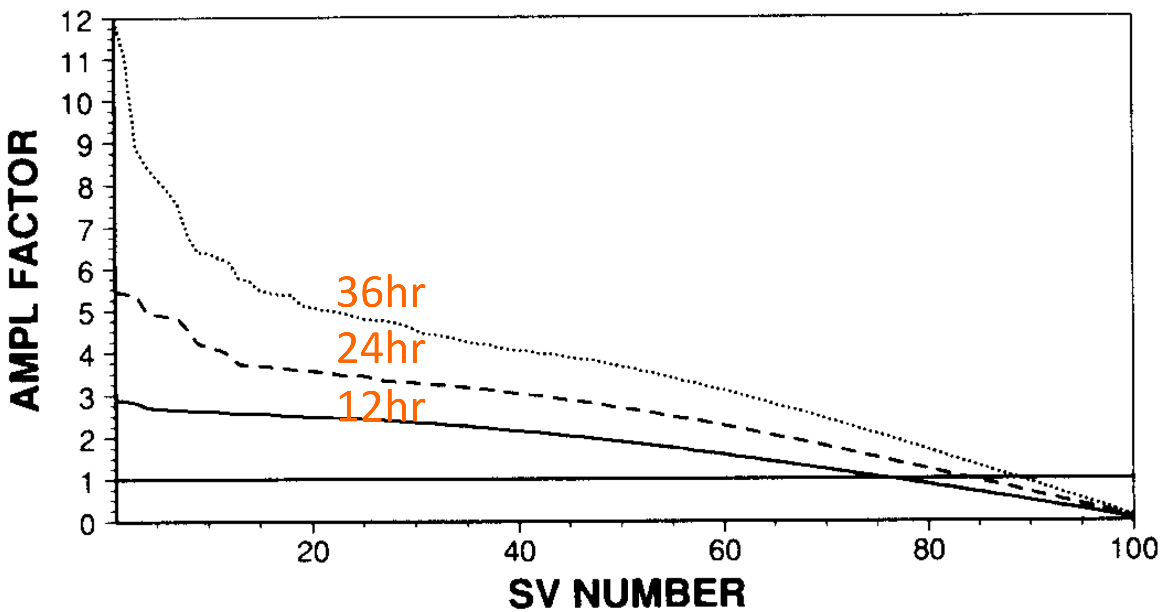


Fig. 2. Amplification factors for the SVs optimized over a 12-h, 24-h and 36-h time interval. The amplification factors are sorted in decreasing order.

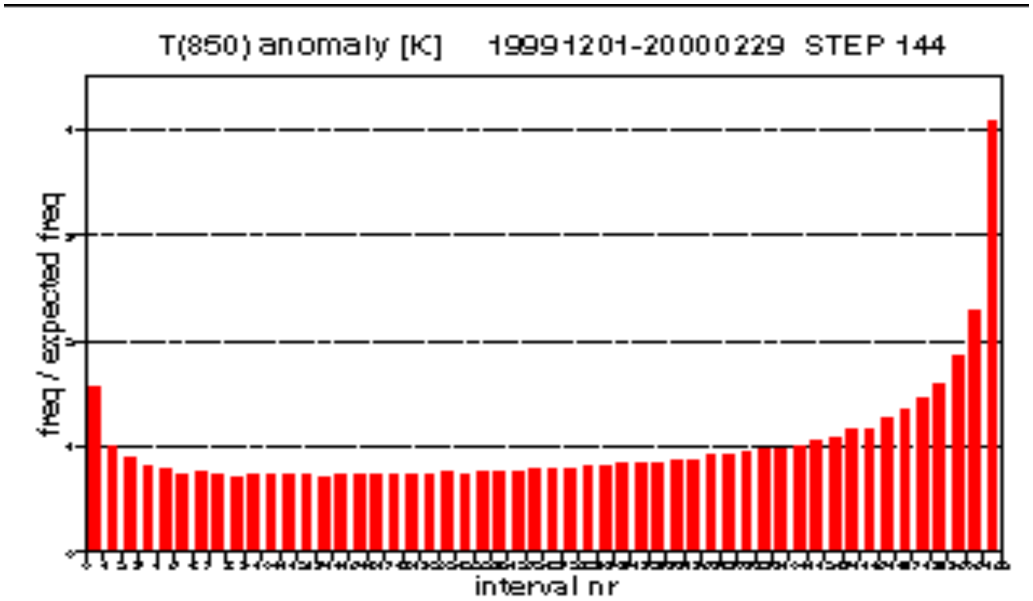
Some (second) thoughts on selective sampling

- Reduced sampling is **ONLY** efficient if one is interested in a few questions only (e.g. sample initial uncertainties dominating forecast error growth defined in terms of total energy during the first 2 days).

- Reduced sampling based on singular vectors (ECMWF) is valid **only** in the **linea**r regime, requires a tangent forward and adjoint model. SV perturbations are metric sensitive.

- Reduced sampling based on breeding vectors (NCEP) is easier to implement, less expensive, but it does not emulate the scale-selective effect of observations during the analysis cycle.

EXAMPLE: distribution bias



Ideally histogram is flat. Wings are over-populated.
Predicted distribution too tight.

3. Fraternal Twins

a natural way to study predictability error growth

Strategic Sampling

has built in bias that might

affect long term predictability

KE spectrum fraternal twin experiments

T180 (1.5°) resolution GCM=Truth
Leaving RESOLVED initial state unchanged
gradually coarsen resolution

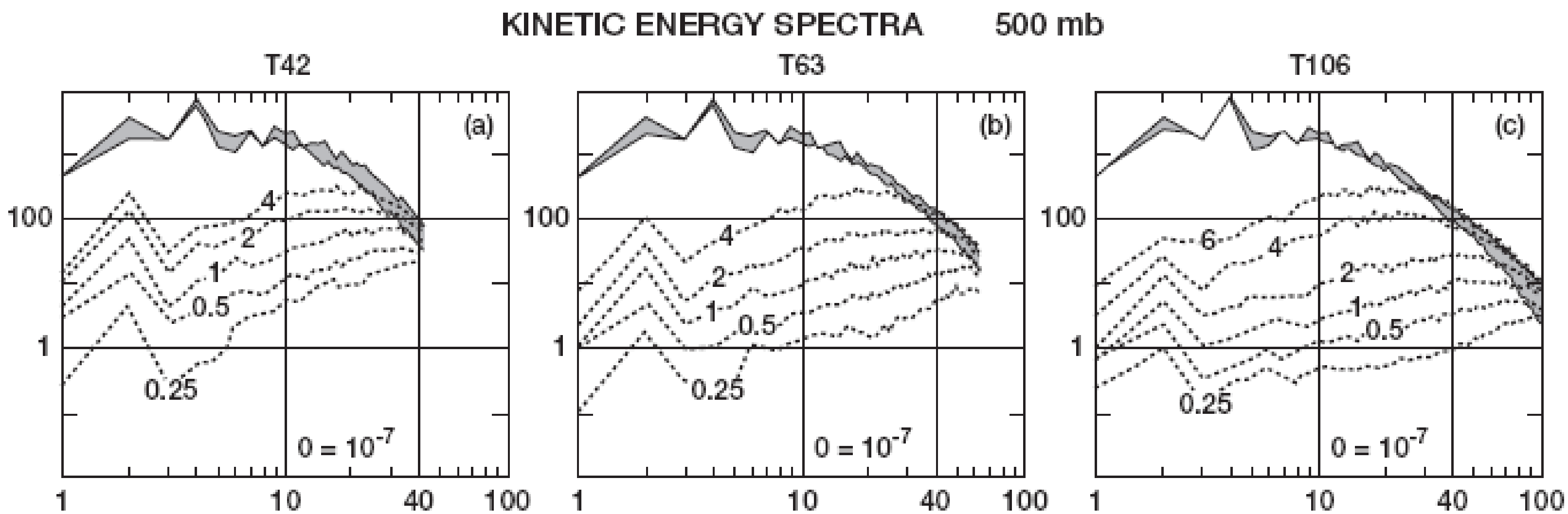
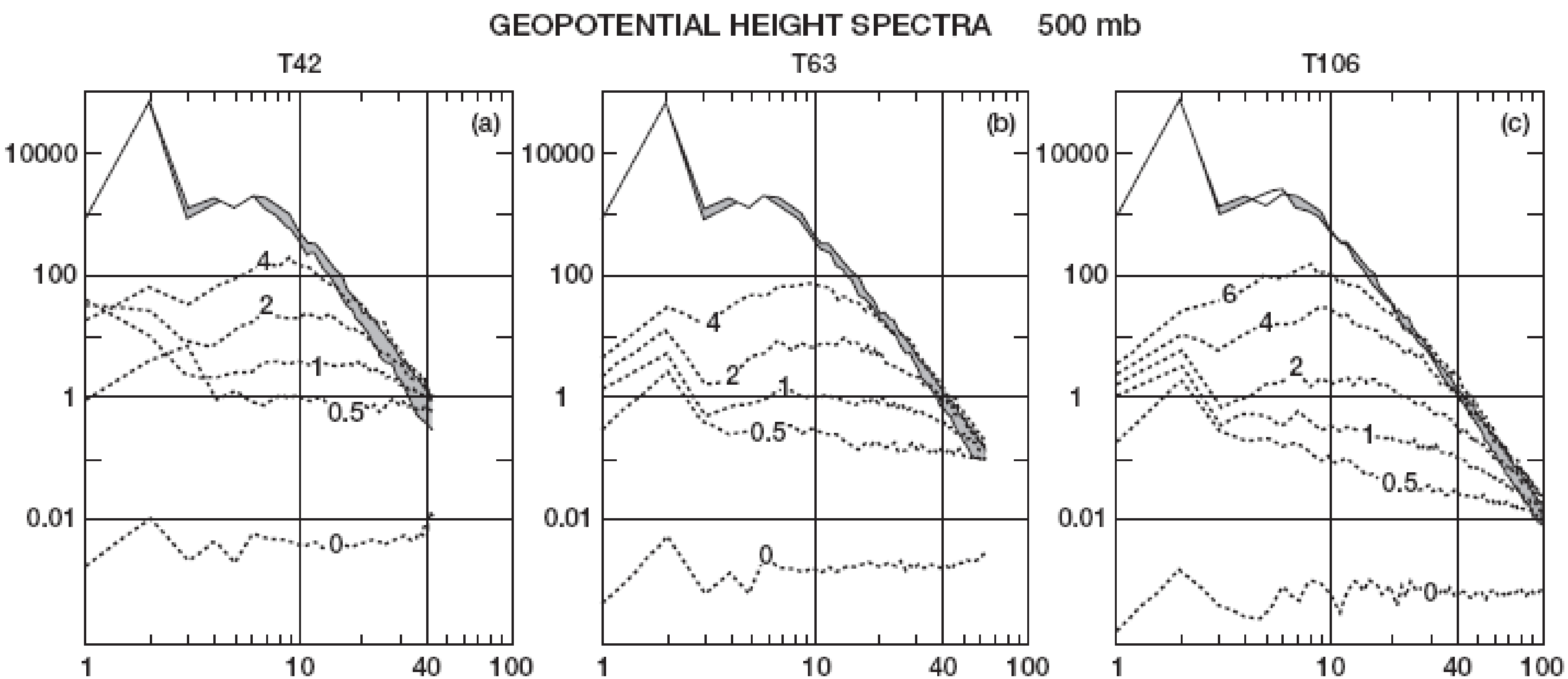


Figure 6: Same as Fig. 2, but for 3 different model truncations. Gray shading shows full spectrum change in 5 days (gray = reduction in energy). Dashed line labeled by time in days shows growth in error as measured by differencing the forecast with T170 run (truth) a) T42 model, b) T63, c) T106.

‘Streamfunction’ Spectrum fraternal twin



Large scales dragged along by synoptic scales

Figure 7: Same as Fig. 6, but for 500mb geopotential.

2D-Turbulence Closure Prediction:

Small scale saturation->inverse cascade of error

Closure theory shows limited predictability because of inverse error cascade

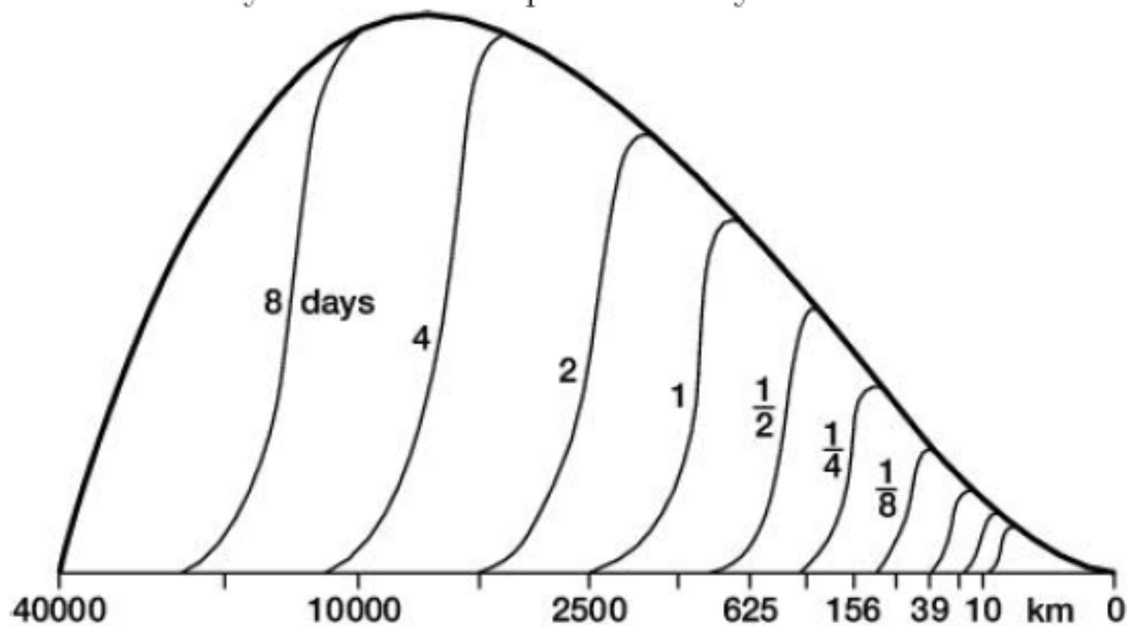
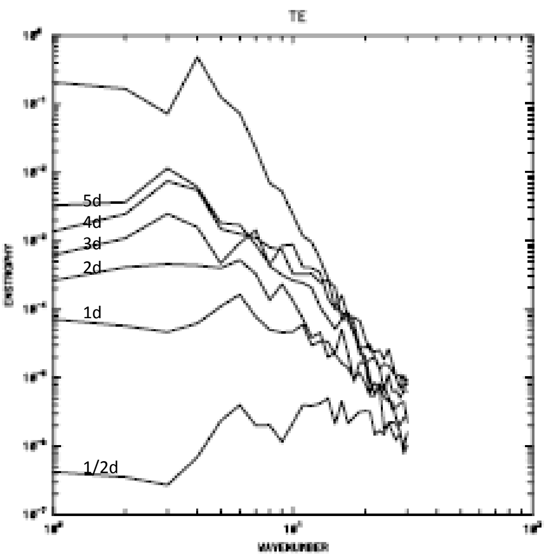


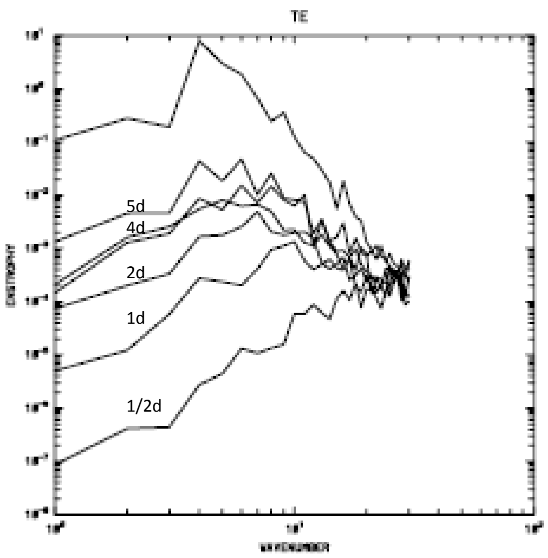
Figure 1: Growth of errors initially confined to smallest scales, according to a theoretical model (taken from a paper by E. Lorenz presented in AIP Conf. Proceedings #106). Horizontal scales on bottom; full atmospheric motion spectrum = upper curve.

Fraternal Twin 2D and Quasigeostrophic Turbulence

Barotropic Model



Quasigeostrophic Model



Both look more like the GCM and less like the saturation inverse cascade picture

4. Singular Vectors and the Inverse Cascade of Error

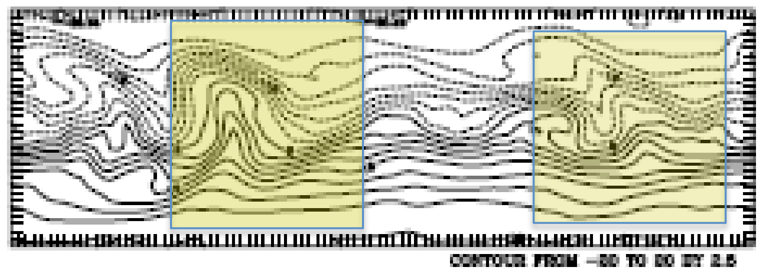
Putting the pieces together

QG basic State and Error Snapshot

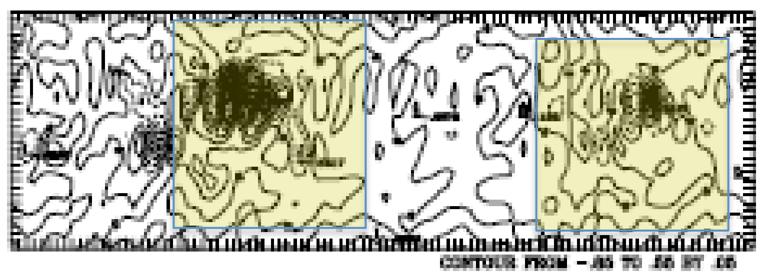
QG Model

Active regions=regions of hydrodynamic strain

Instantaneous
Streamfunction



Instantaneous
Fraternal Twin
QGPV error field

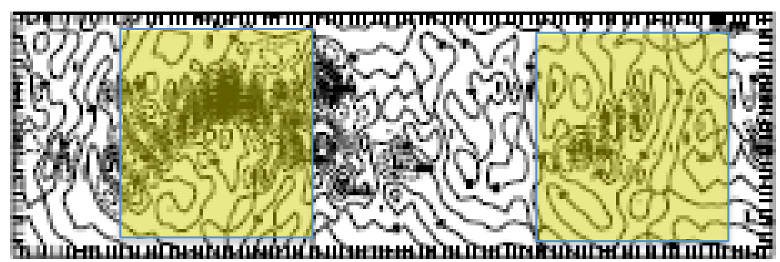


Error vs Leading SV

Active regions also regions of large amplitude in leading singular vector

Instantaneous
Fraternal Error
In QGPV

Leading Energy Norm
Singular Vector
QGPV field



Lagrangian PV Dynamics means
Fluid Strain equates to Phase Space Strain

Conclusions

- Use of Adjoint Sensitivity Analysis in Ensemble Weather Prediction (evidence of Marchuk's strong and broad influence)
- Probabilistic /Hydrodynamic interpretation of Singular Vectors
- Saturation/Inverse Cascade Ideas require Modification
- Singular Vectors for Analysis of Information loss

The End

Thank-you

and

Questions?

EXTRA SLIDES

Nonlinear terms can only conservatively exchange energy Spectral properties can be gotten by dimension analysis

$$[E(k)] = L^3T^{-2}$$

$$[\varepsilon] = L^2T^{-3} \quad \text{and} \quad [k] = L^{-1}$$

so

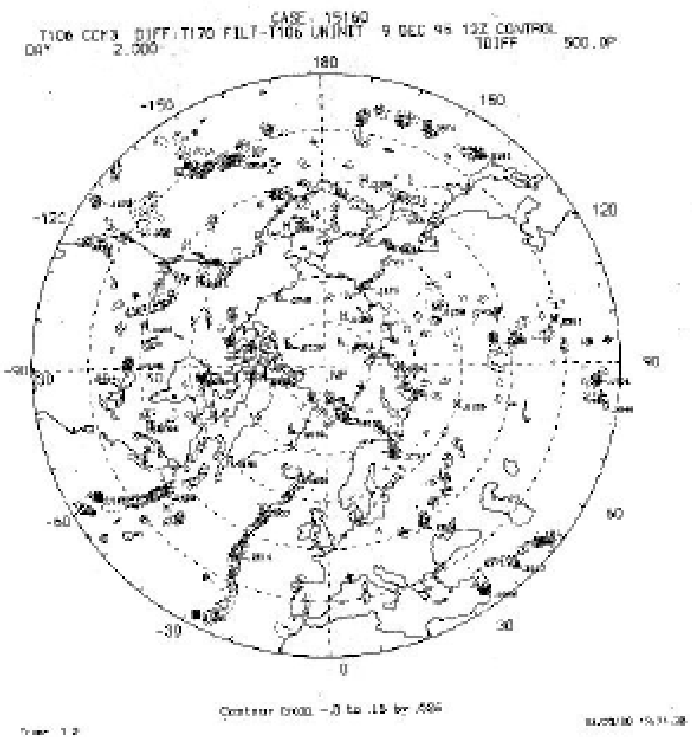
$$E(k) = C_1\varepsilon^{2/3}k^{-5/3}$$

for some universal C_1 of order 1.

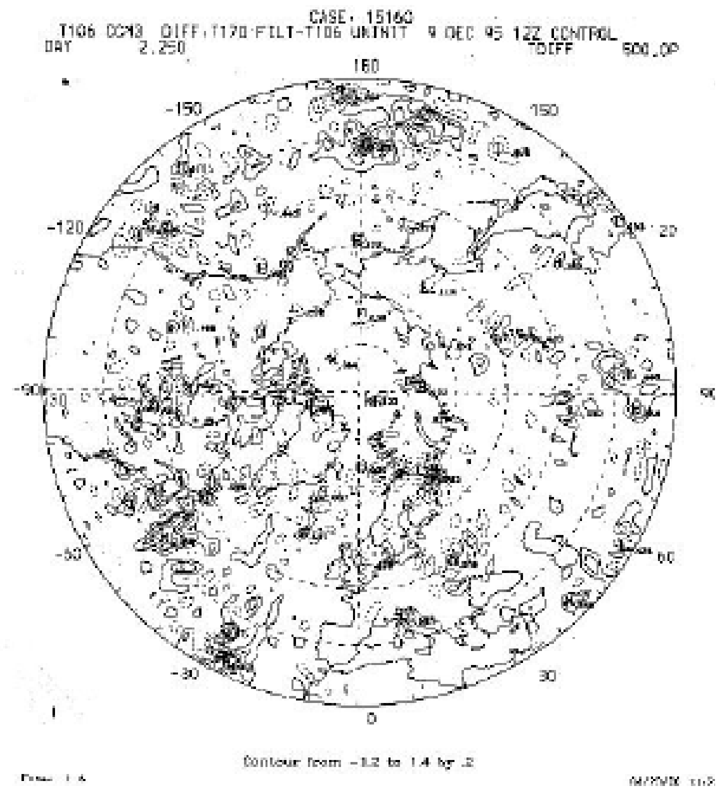
Real Model Error-Random Variety

Random Errors - Resolution and Flat Topography

T error at day 0



T error at day 1/4



Decomposition in space at t=0

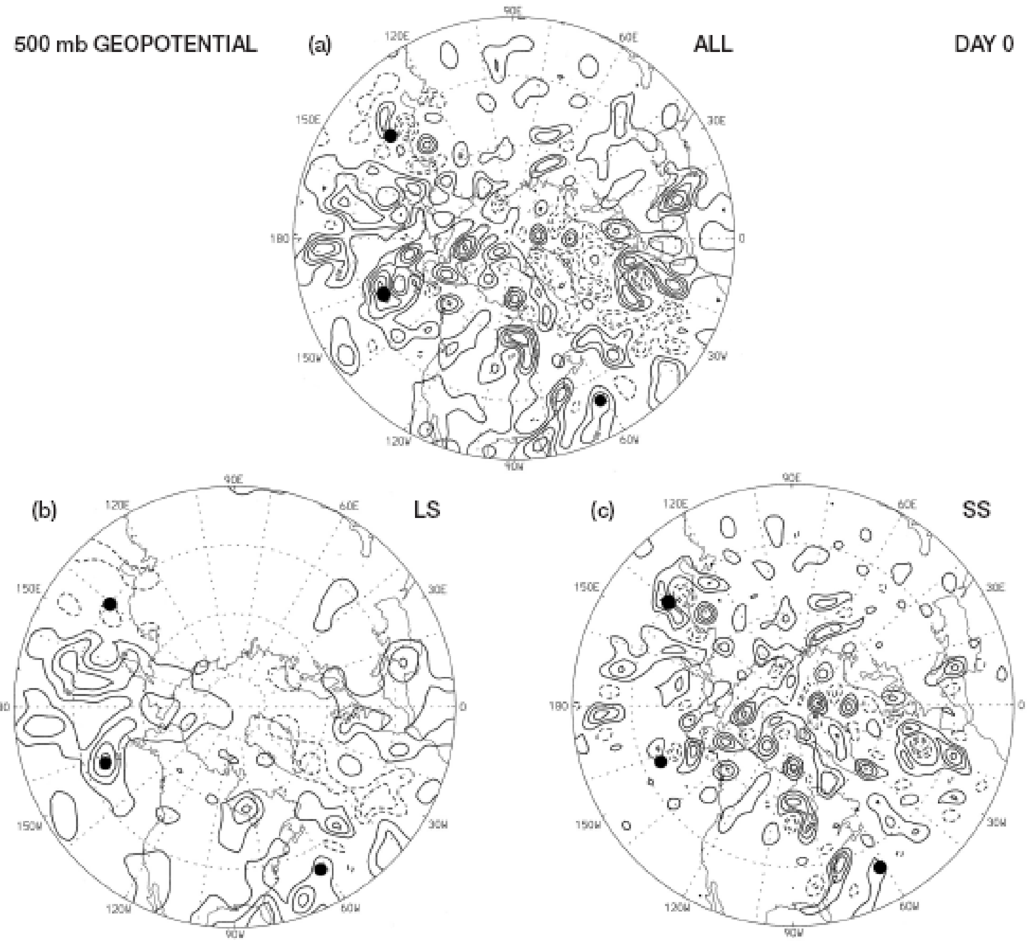


Figure 3: Difference field of 500 mb geopotential at initial time for 1 pair of ensemble members shown in Fig. 2.
Contour interval = 10m; solid = positive difference; dashed = negative difference; black dots are locations referred to in text.

Growth after one day

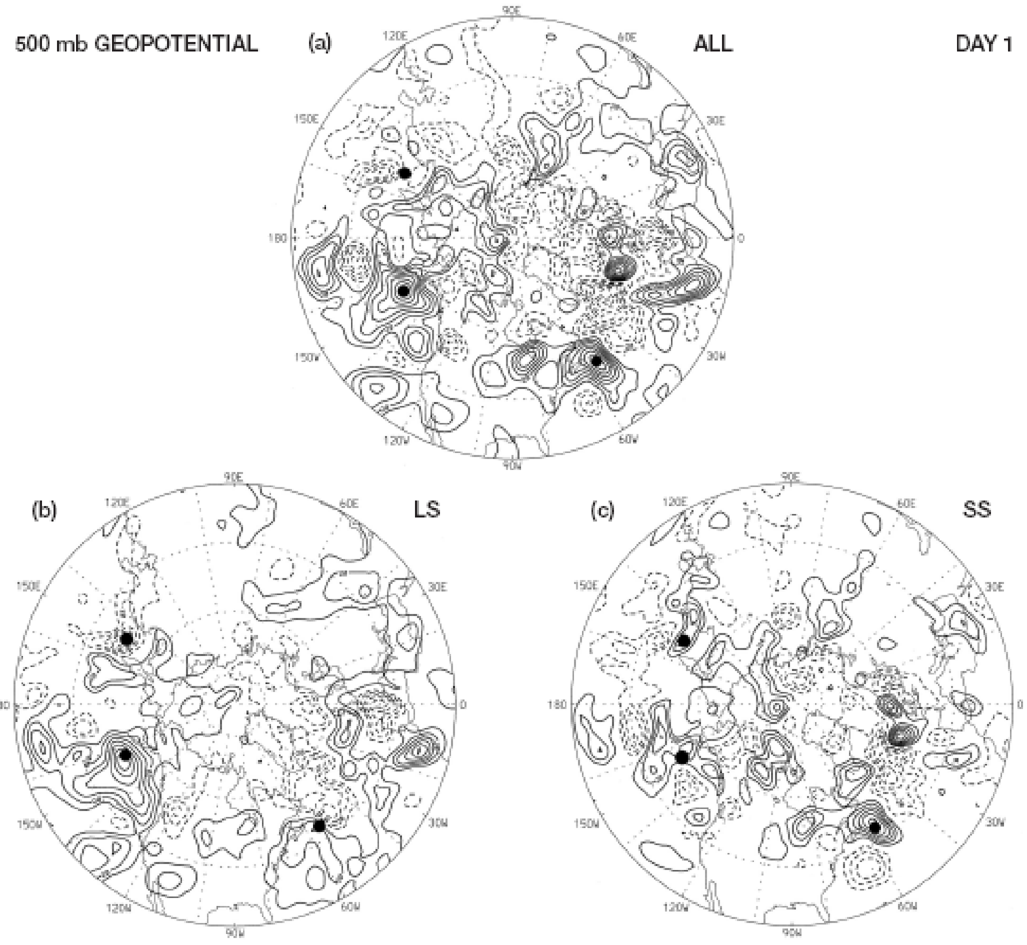


Figure 4: Same as Fig. 3, but for 1 day forecast.

Spatial growth at 3 days

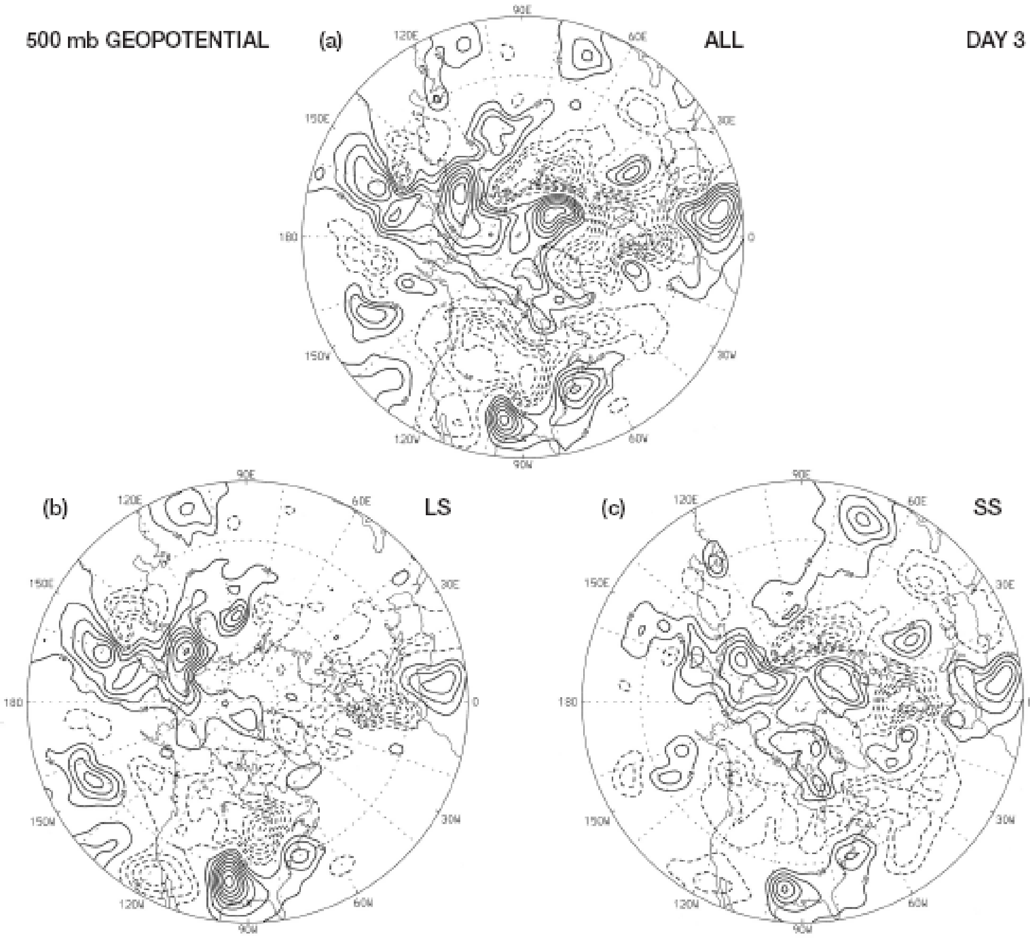


Figure 5: Same as Fig. 3, but for 3 day forecast. CI = 20m

Error Growth is ‘almost’ linear- use this

Singular Vectors, Generalized stability Pseudo-eigenanalysis

- Linear theory: singular vectors
- $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}, t)$
- $\vec{x}(0) = \vec{X}_0$ or $\vec{x}(0) = \vec{X}_0 + \varepsilon \vec{X}_1$
- 2 solutions $\vec{x}_0(t)$ and $\vec{x}_1(t)$; Let $\vec{z}(t) = \vec{x}_1(t) - \vec{x}_0(t)$.
- Then $\frac{d\vec{z}}{dt} \cong A(t)\vec{z}$ with $A \equiv \frac{\partial \vec{F}}{\partial \vec{x}}(\vec{x}_0, t)$
- and $\vec{z}(0) = \vec{X}_1$. $\vec{z}(t) = R(t)\vec{z}(0)$.
- SV's are $\cong$leading eigenvectors of $R(t)R^t(t)$

By Rayleigh-Ritz also are vectors with maximal growth in energy norm
BVs build the initial error covariance into the norm

Outline

- Review the basics of 3D vs 2D turbulence and its QG partner
- Constraints and self-consistency of QG
- Beyond QG to Nastrom-Gage range
- Some high resolution studies
- How to break QGT and get N-G spectrum
- Potential predictability and modeling implications and other stuff going on in the atmosphere

Conclusions

- Cascade paradigm inappropriate for predictability
- Singular vector growth
- Modal growth in QG, not in 2D
- Threshold nonlinearity in 'real' models

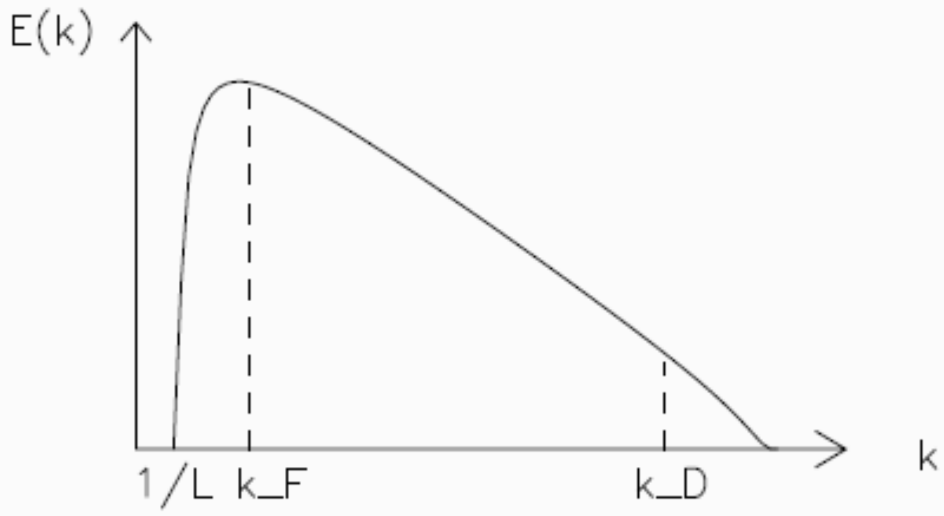
Two questions of a mathematical nature:

Is there a maximum principle for predictability?

Can state dependent error growth bounds be useful?

Recall the cascade concept of 3D turbulence

Kolmogorov (1941) theory



For 3D, statistically steady, homogeneous, isotropic turbulence, in an **inertial range**:

At wavenumber k, the only dimensional quantities are the energy throughput ε and k itself.

QG simple enough to dynamically analyze predictability

Singular Vectors, Generalized stability Pseudo-eigenanalysis

- Linear theory: singular vectors
- $\frac{d\overrightarrow{x}}{dt} = \overrightarrow{F}(\overrightarrow{x}, t)$
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- SV's are $\cong$leading eigenvectors of $R(t)R^t(t)$

By Rayleigh-Ritz also are vectors with maximal growth

Atmospheric spectrum inspired by two-dimensional turbulence

Some time scales:

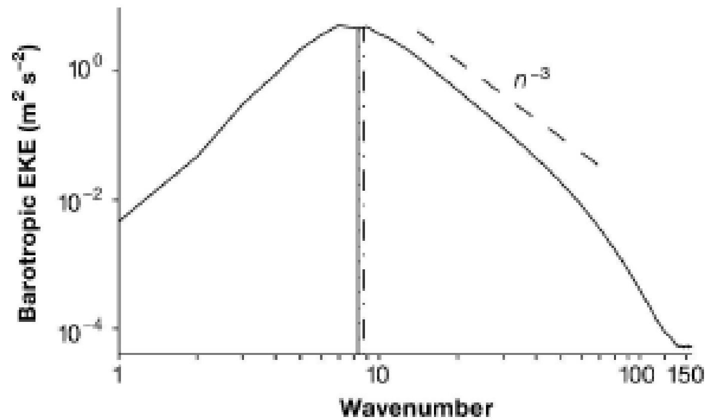
$T_{eddy} \sim (E(k)k^3)^{-1/2}$

$T_{rossby} \sim k/\beta$

Rhines scale
Length at which:

$T_{eddy} = T_{Rossby}$

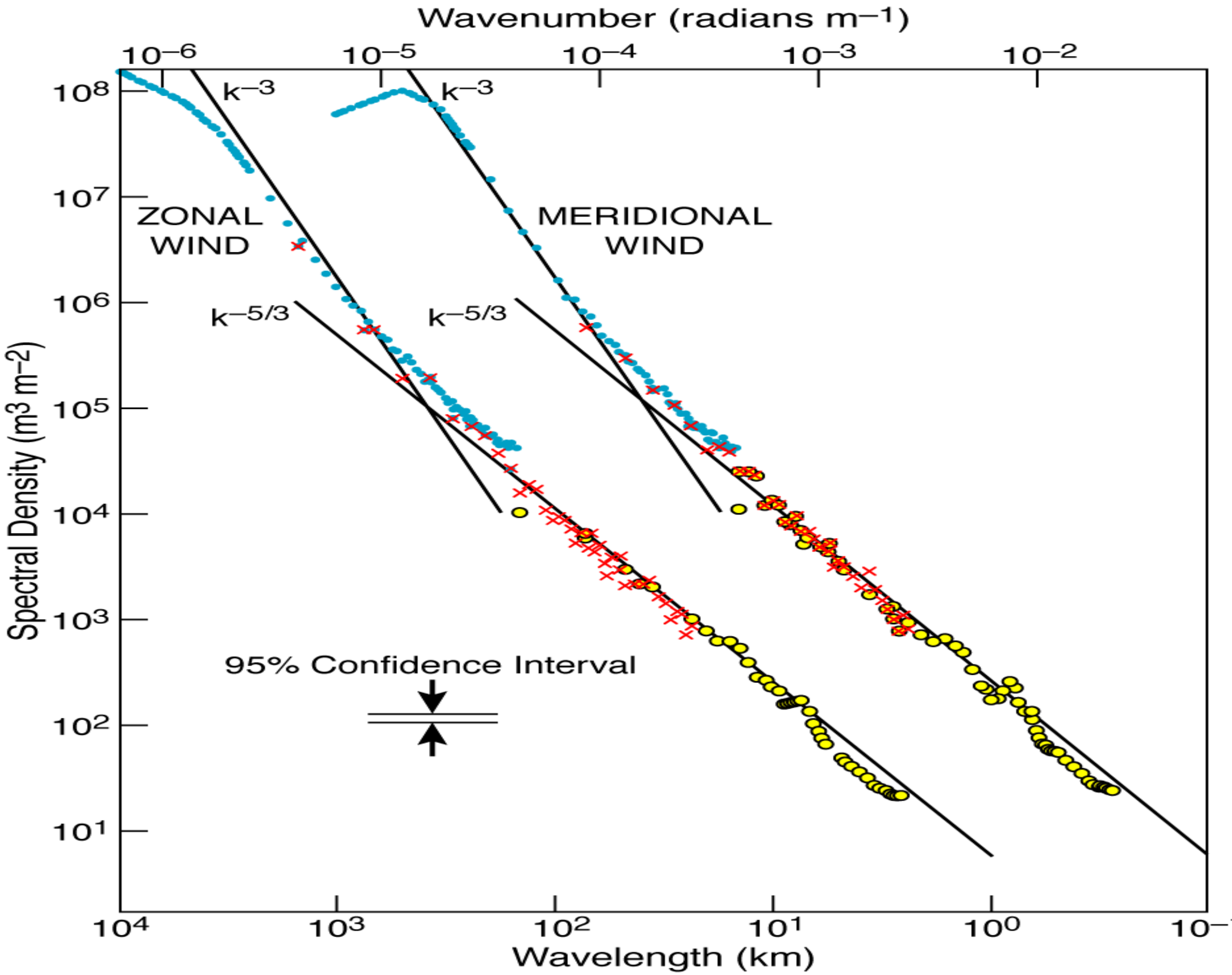
-5/3 range $T_{eddy} \sim k^{-2/3}$
-3 range $T_{eddy} \sim$ const



Wavenumbers near 10 correspond to both the Rhines scale and the injection scale. Energy cascade to Large scales is inhibited by Rossby wave motion .

Few Rossby wave resonances and few wavenumbers

Can we explain the Nastrom & Gage Spectrum?

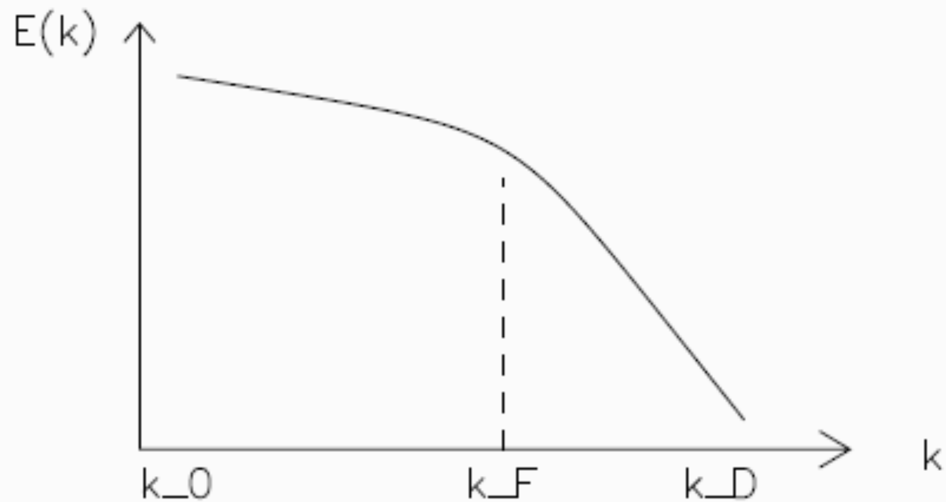


How do things change in 2D? Nonlinear terms conservatively exchange both energy and enstrophy

Two dimensional turbulence

In 2D turbulence we have another conservable quantity, the enstrophy, and therefore a cascade of enstrophy η.

Typically energy now cascades upscale while enstrophy cascades downscale.



In 2d turbulence the enstrophy conserving range gives:

$[E(k)] = L^3T^{-2}$

$[\eta]=T^{-3}$ and $[k]=L^{-1}$

So $E(k)= C_2\eta^{2/3}k^{-3}$

How do we decide direction? Use a variant of Fjortoft Theorem

Energy upscale, enstrophy downscale (mostly)

Let

$$E = \int E(k)\,dk \quad \text{and} \quad Z = \int k^2 E(k)\,dk$$

Suppose energy is initially concentrated near wavenumber k_1 and subsequently spreads out, so that

$$\frac{d}{dt}\int (k-k_1)^2 E(k)\,dk > 0$$

The fact that E and Z are conserved (neglecting viscosity) implies

$$\frac{d}{dt}\left(\frac{\int kE(k)\,dk}{\int E(k)\,dk}\right) < 0$$

The Spectral Enstrophy transfer is then constrained as follows:

Similarly, assuming

$$\frac{d}{dt}\int (k^2 - k_1^2)^2 E(k)\, dk > 0$$

implies

$$\frac{d}{dt}\left(\frac{\int k^2 Z(k)\, dk}{\int Z(k)\, dk}\right) > 0$$

On to QGT : Summary of Mid-latitude QG theory

beta plane (*dx=a cos(Φ)dλ, dy=a dΦ*)

$$u_t - fv + \phi_x = -(uu_x + vu_y + \omega u_p),$$
$$v_t + fu + \phi_y = -(uv_x + vv_y + \omega v_p),$$
$$\phi_{pt} + S(p)\omega = -(u\phi_x + v\phi_y + \omega\phi_{pp} + \frac{\omega}{p}(1-\kappa)\phi_p),$$
$$u_x + v_y + \omega_p = 0.$$

We non-dimensionalize the equations to isolate the scales we are interested in: $(x, y) = L(x', y'), p = P_0 p', t = f_0^{-1} t', (u, v) = U(u', v'), \omega = \frac{U}{L} P_0 \omega'$, where the quantities U, L, P_0, f_0 are 'typical' values of the wind velocity, horizontal length scale, pressure depth and Coriolis parameter of midlatitudes weather systems. If we rewrite the governing equations in terms of the prime (non-dimensional) quantities ,we get:

$$u_t - v + \phi_x = -R_o(uu_x + vu_y + \omega u_p) + \frac{\beta L}{f_0} yv,$$
$$v_t + u + \phi_y = -R_o(uv_x + vv_y + \omega v_p) - \frac{\beta L}{f_0} yu,$$
$$\phi_{pt} + B\omega = -R_o(u\phi_x + v\phi_y + \omega\phi_{pp} + \frac{\omega}{p}(1-\kappa)\phi_p),$$
$$u_x + v_y + \omega_p = 0,$$

Scaled equations have several small terms

- Rossby number $R_o = U/(f_0 L)$ is ~0.1
- If L is restricted so that L<<a then $\beta L/f_0$ is also ~0.1
- Burger number $B = S(p)P_0/(f_0 L)^2$ is order 1
- Perfect for asymptotic expansion in R_o
- Expand all dependent variables in a series in powers of R_o and match powers

At order zero we get a linear system constant coefficients

$$\begin{aligned} u_t^0 - v^0 + \phi_x^0 &= 0, \\ v_t^0 + u^0 + \phi_y^0 &= 0, \\ \phi_{pt}^0 + B\omega^0 &= 0, \\ u_x^0 + v_y^0 + \omega_p^0 &= 0. \end{aligned}$$

The solution can be given as a superposition of inertia gravity waves and vortical modes with frequencies

$$\sigma_{\pm} = \pm(f^2 + gH_0(k^2 + l^2))^{1/2}$$

for inertia gravity waves and $\sigma_0 = 0$ for vortical modes

At 1st order restrict motion to slow time scale: Nonlinear QG equations

$$q_t + J(\psi, q) = 0,$$

$$q = \nabla^2\psi + (f^2/S)\psi_{pp} \equiv L\psi$$

$$\psi_p = 0$$

$$p = 0, p_s$$

Similar to two–dimensional non-divergent governing equations. Isomorphic if variation in p is ignored and:

$$q \equiv \nabla^2\psi$$

Note: We also get diagnostic equation for divergence
The QG omega equation $L\omega = F$

$$\nabla^2\omega + (f^2/S)\omega_{pp} = F(\psi)$$

Quasi-slow manifold and QG turbulence

$$q_t + J(\psi, q) = 0,$$

$$q = \nabla^2\psi + (f^2/N^2)\psi_{zz} \equiv L\psi$$

Fjortoft Constraints

$$\text{Total Energy } E = -\int(\psi L\psi)dV$$

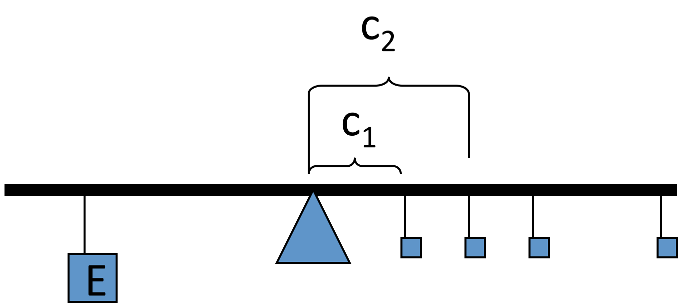
$$\text{Pot. Enstrophy } EN = \int(L\psi)^2 dV$$

$$L\varphi_n = -c_n^{\,2}\varphi_n$$

$$\psi = \sum a_n\varphi_n,\ E = \sum|a_n|^2 c_n^{\,2}$$

$$\text{Let } e_n \equiv |a_n|^2 c_n^{\,2}$$

$$E = \sum e_n \ \text{ and } EN = \sum c_n^{\,2} e_n$$



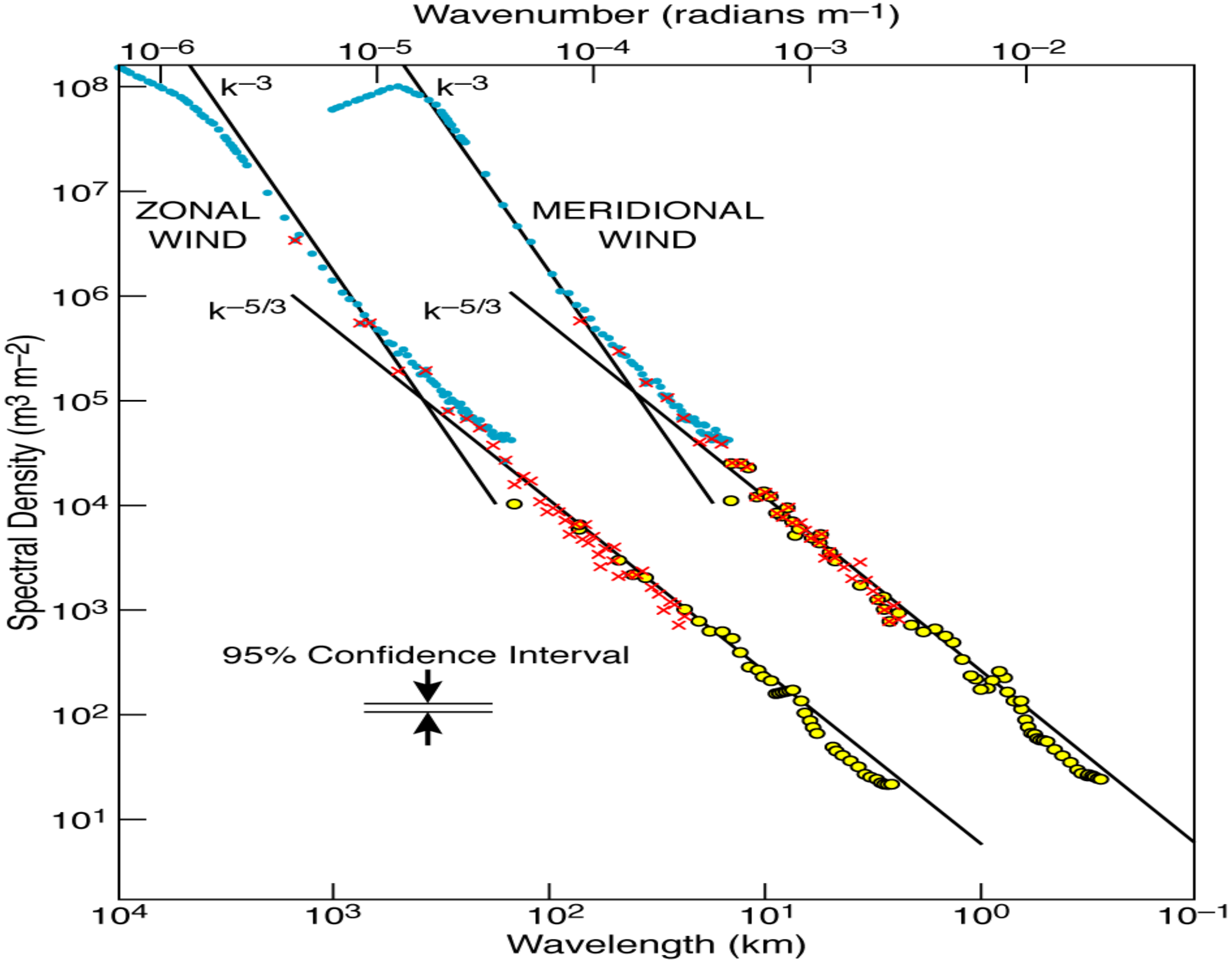
Cannot maintain balance if
Energy moves toward large n
Energy cannot be cascaded
to small scale

Implications of QG Turbulence

- Potential Vorticity analog of 2D vorticity
- Potential Enstrophy cascaded to small scale with zero flux of total energy (KE+APE)
- Total energy spectrum $\alpha\ k^{-3}$
- Ro(L)= $(\text{Enstrophy})^{1/2}/f$= constant
- Ri(L)-> ∞ as L->0
- No new instabilities
- QGT unbreakable down to 3D isotropic scales

Energy containing eddies QG ->all scales QG

How can we then explain the Nastrom & Gage Spectrum?

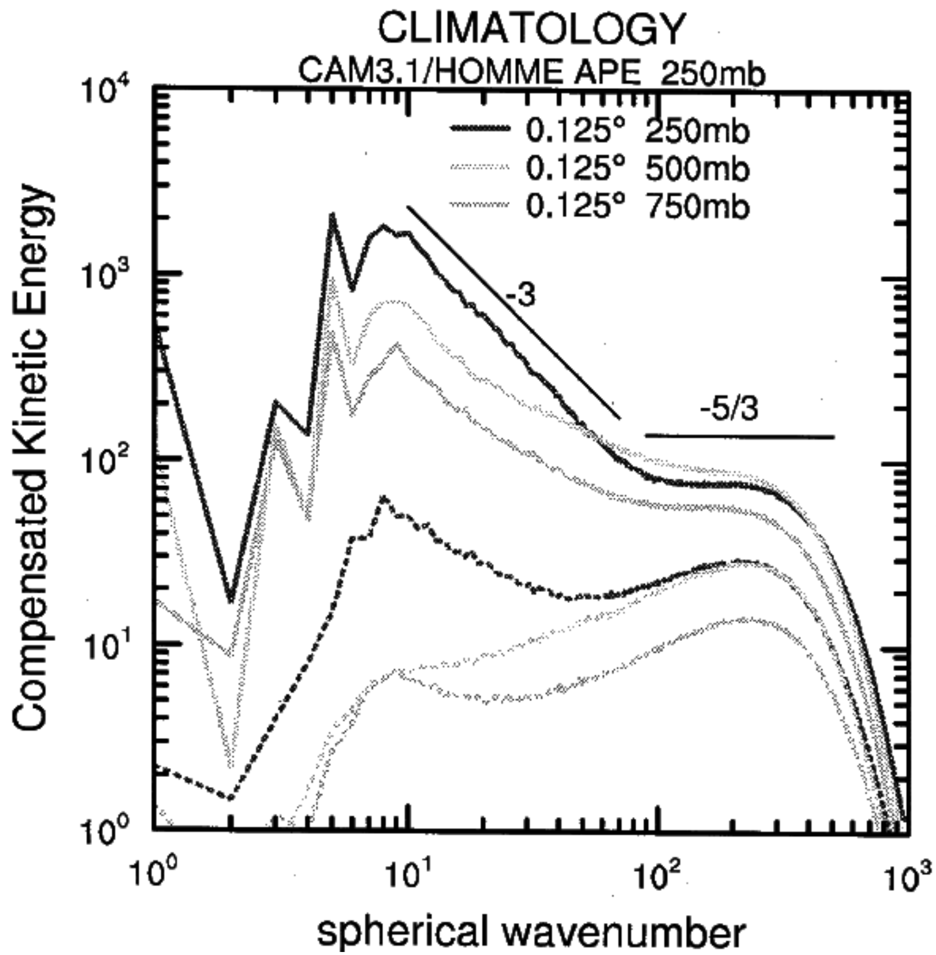


Some possibilities (that could work)

- Inverse cascade of balanced turbulence due to injection of energy by small scale convection (Gage, Lilly)
- N^2 scaling (Kimura and Herring)
- Stratified Turbulence (Lindborg)
- Surface geostrophic dynamics at the tropopause (Tulloch and Smith)

Height dependence of the spectrum (SGT?)

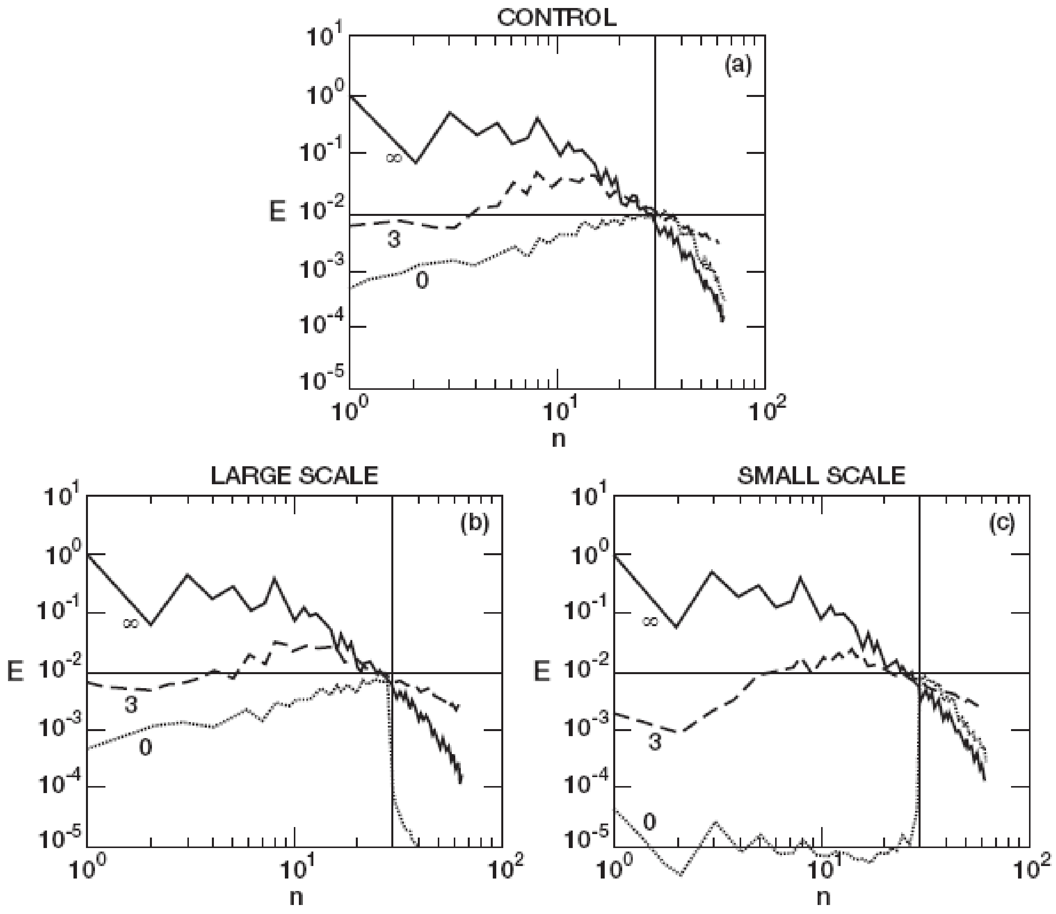
Compensated
Spectrum
$E(k) \times k^{5/3}$



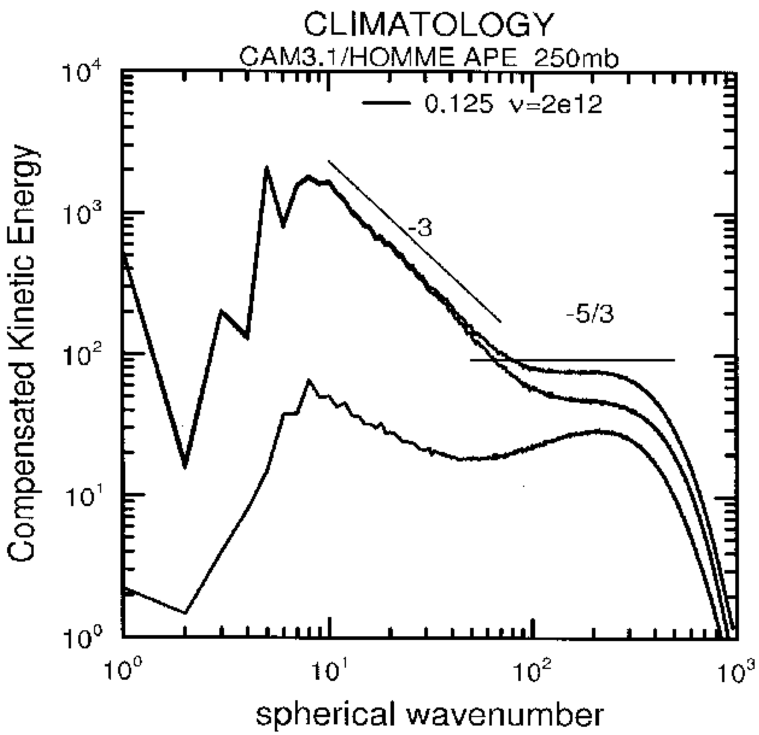
Links to limited area modeling

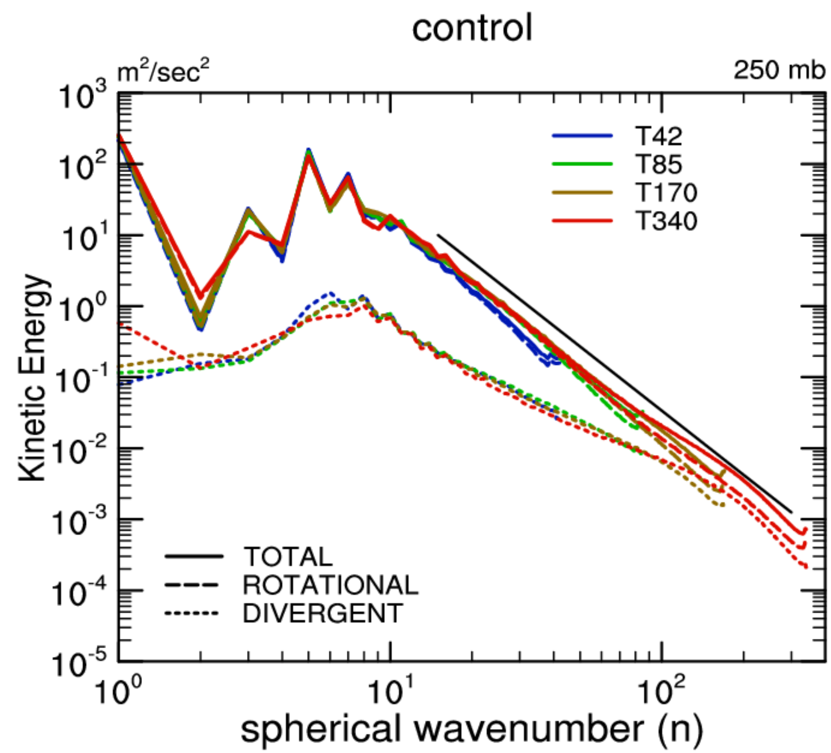
Examine band limited Predictability.

Only permit errors in Large/Small scales

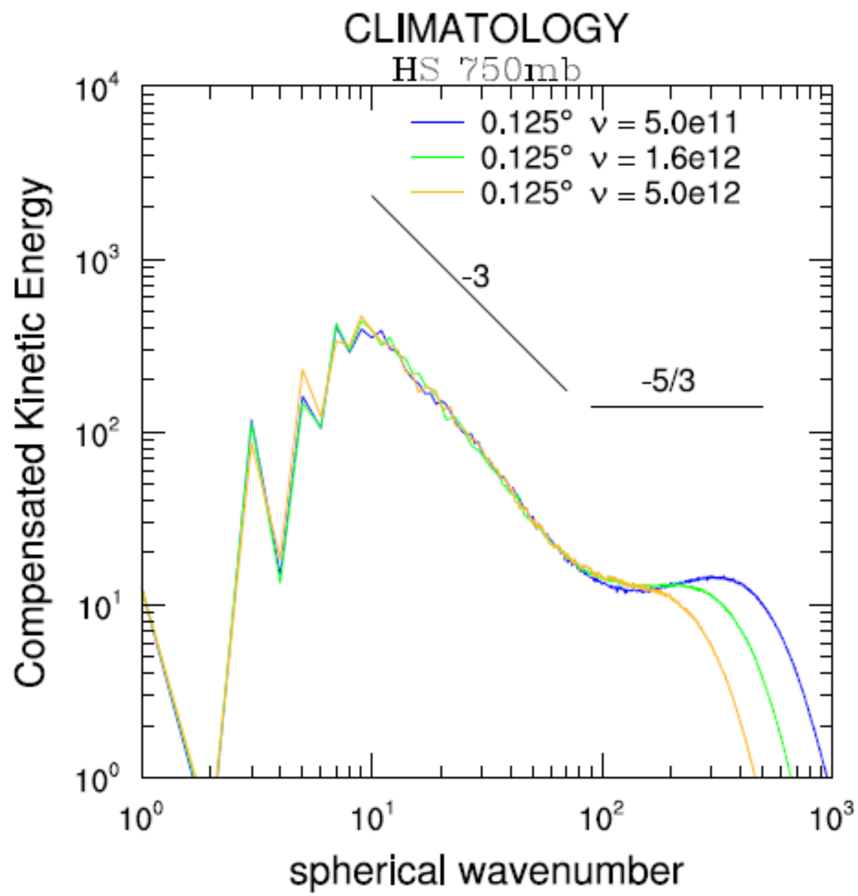


HOMME and Spectral results (Taylor and Williamson)





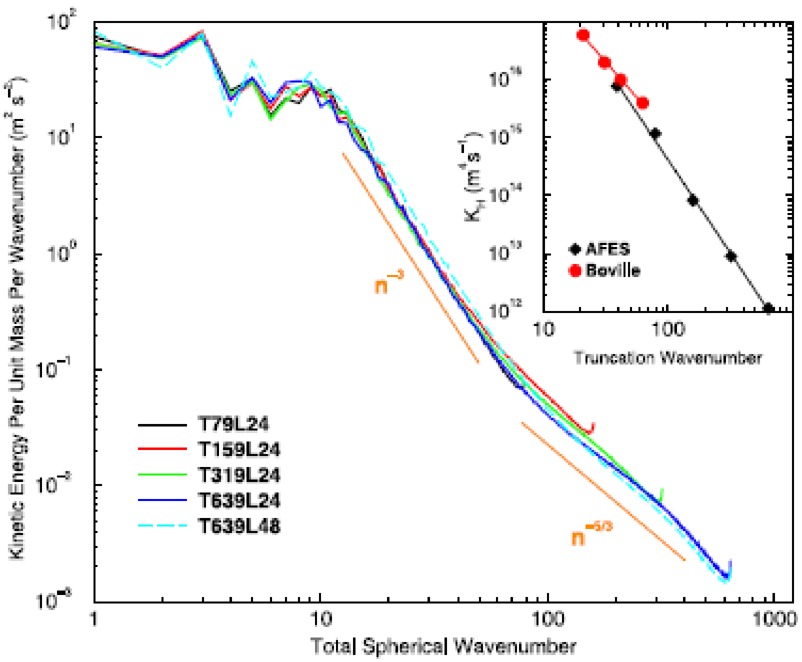
Dry hydrostatic dynamics
Held-Suarez forcing



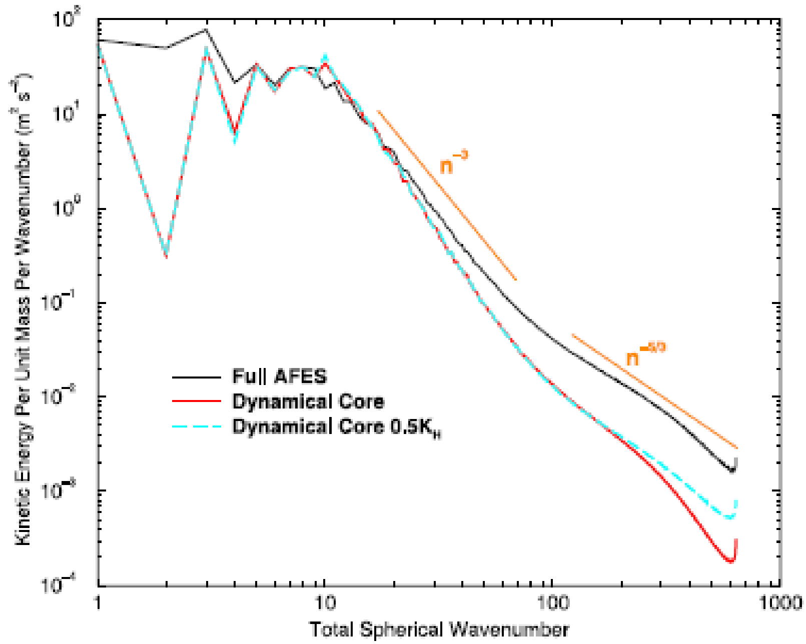
Bottleneck?

Earth Simulator

Takahashi et al tuned viscosity

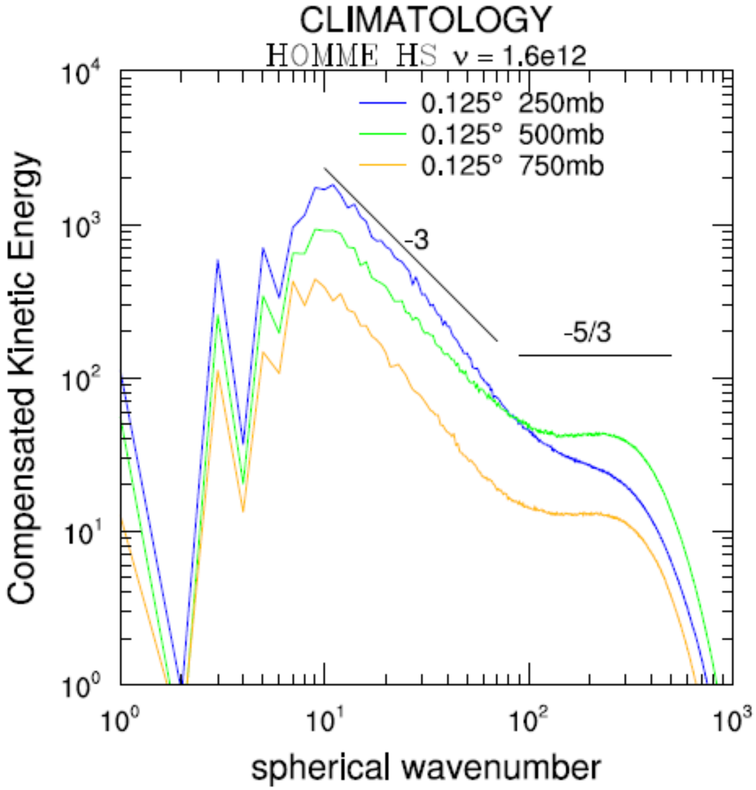


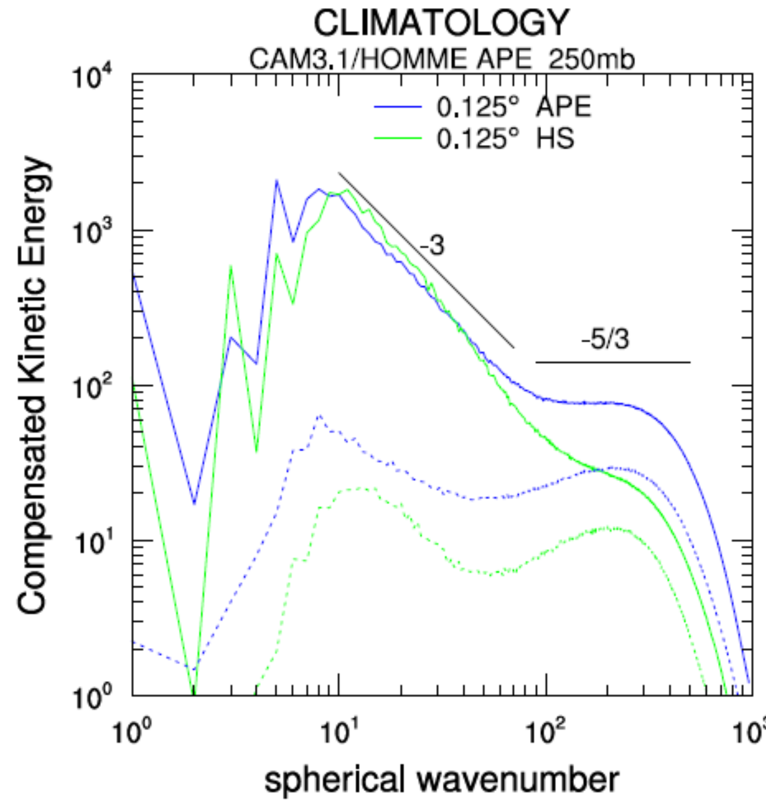
Full moist physics



Comparison with dry dynamics

HS forcing height dependence and comparison with aqua planet





QGT no longer dominant for L<100km
What is going on?

Restrict motion to slow time scale: Nonlinear QG equations

$$q_t + J(\psi, q) = 0,$$

$$q = \nabla^2\psi + (f^2/S)\psi_{pp} \equiv L\psi$$

$$\psi_p = 0$$

$$p = 0, p_s$$

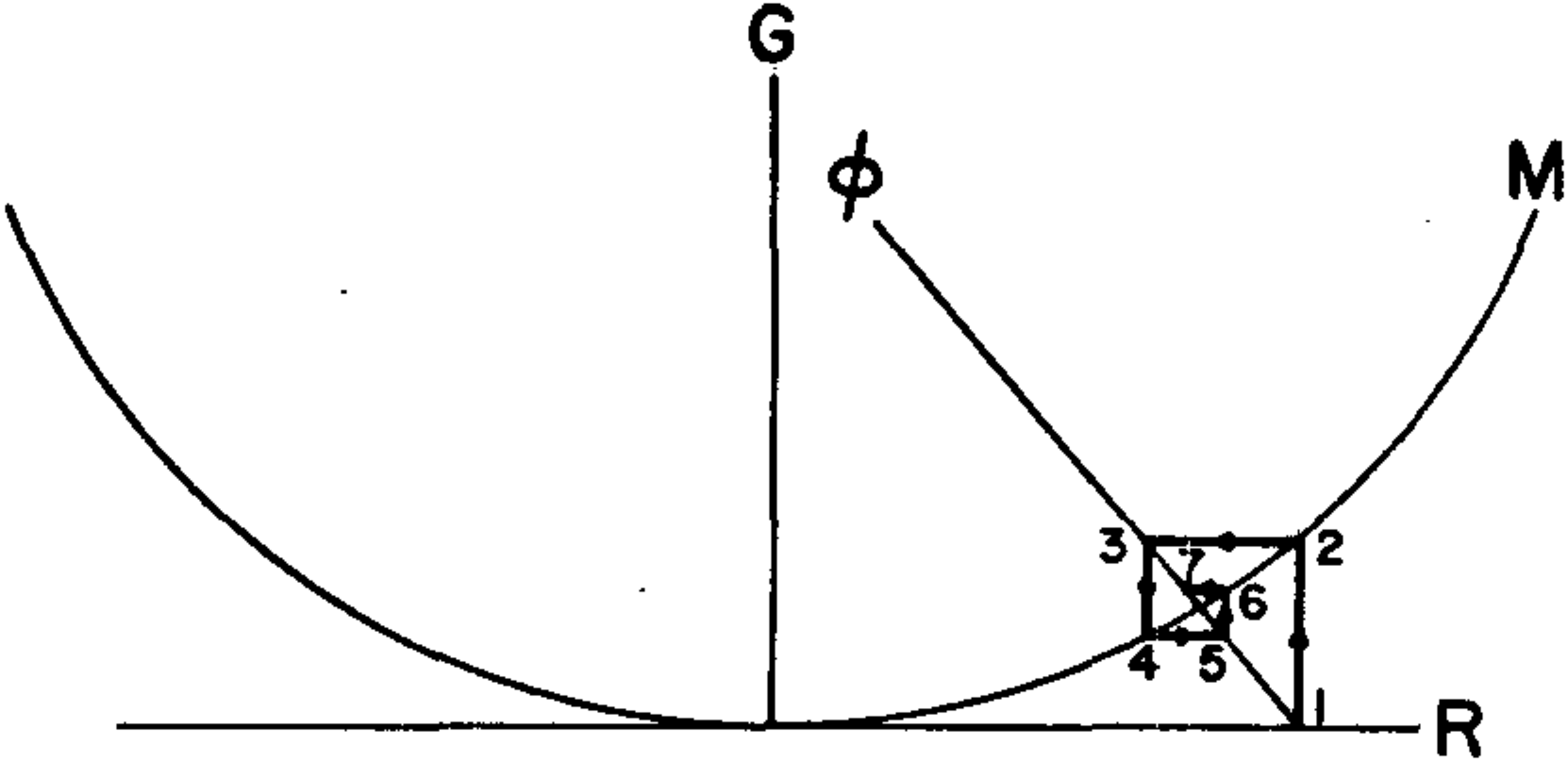
Similar to two–dimensional non-divergent governing equations. Isomorphic if variation in p is ignored and:

$$q \equiv \nabla^2\psi$$

NB: We also get diagnostic equation for divergence
The QG omega equation *Lω=F*

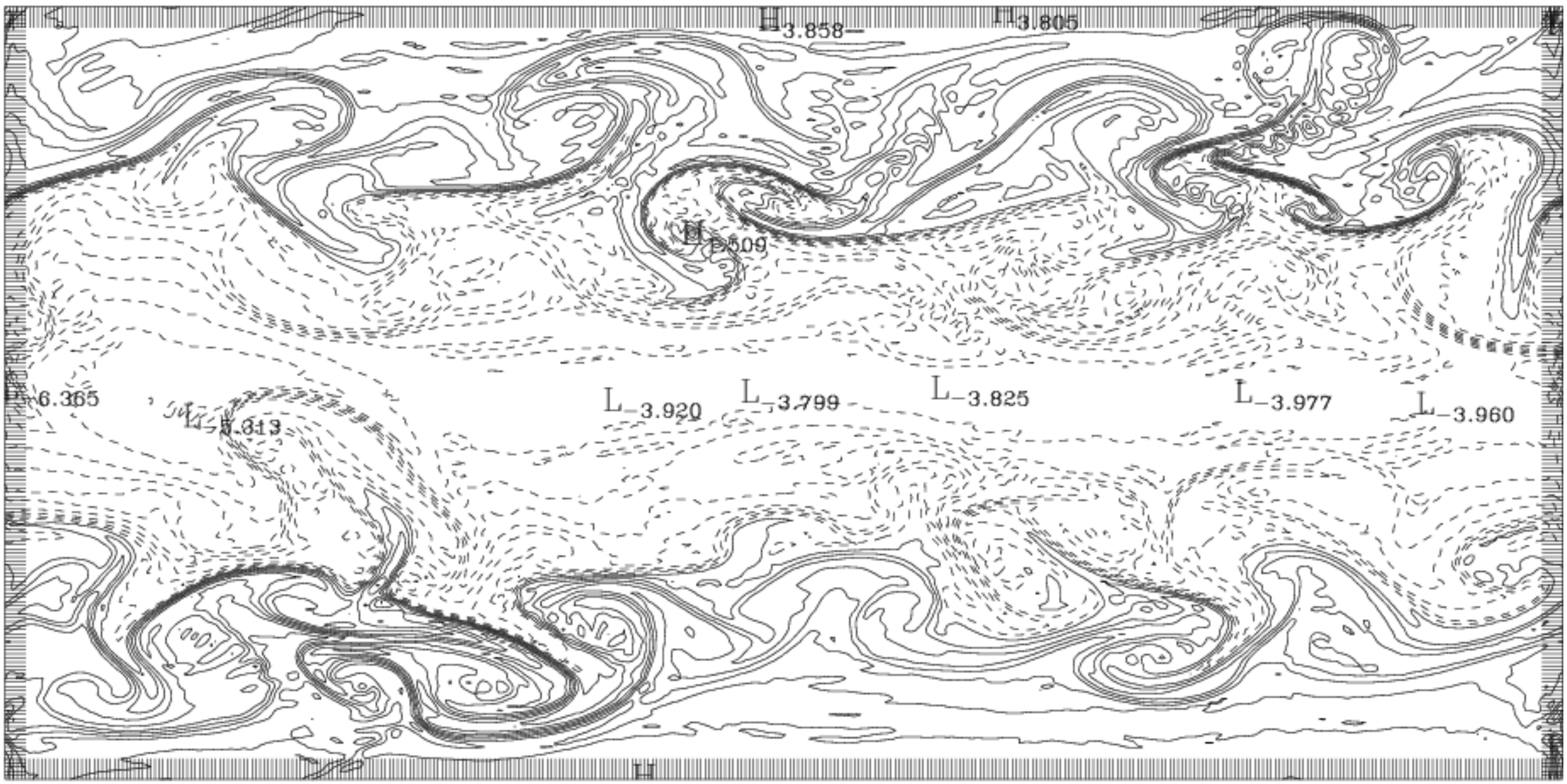
$$\nabla^2\omega + (f^2/S)\omega_{pp} = F(\psi)$$

Hypothetical Curved Slow Manifold



Question: How is G(R) spectrally distributed?

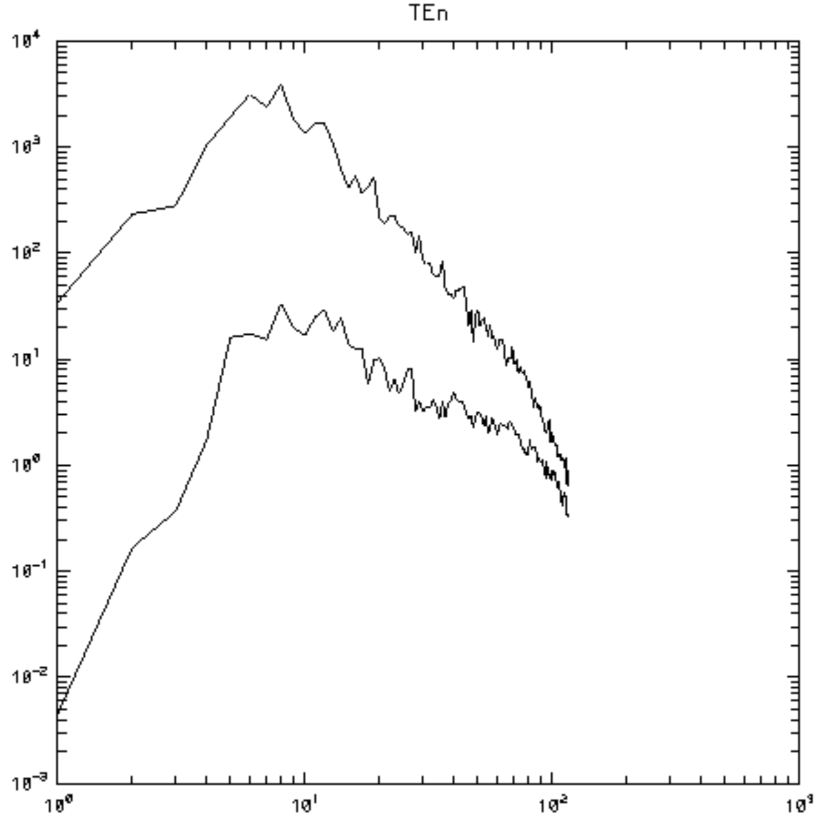
QG Turbulence
Potential Vorticity



CONTOUR FROM −6 TO 5.5 BY .5

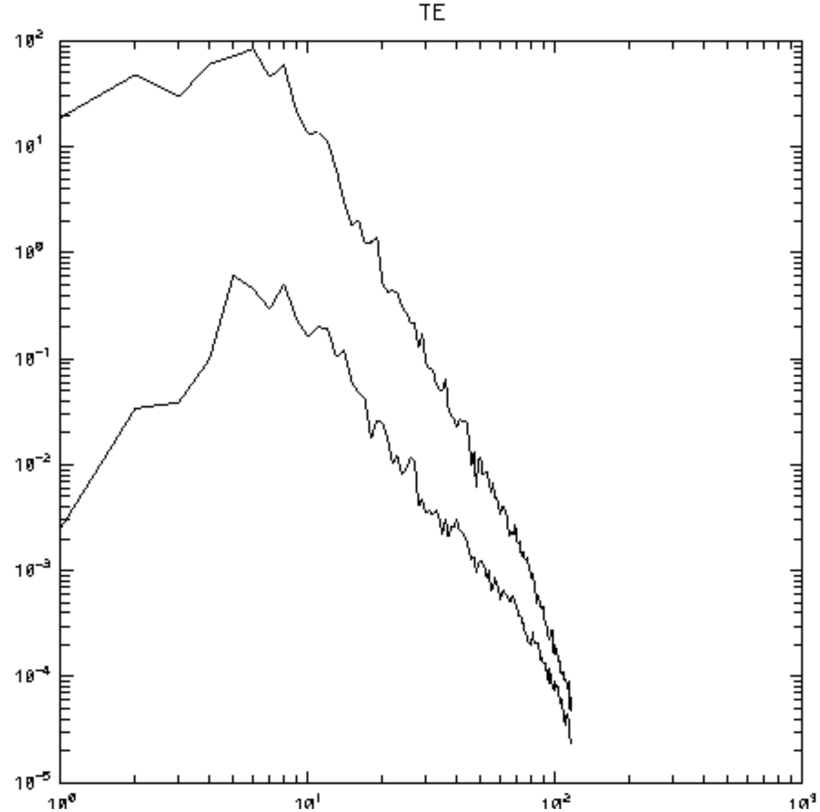
Balanced Divergent Wind spectrum from QG

Rot
Div
Ens



wavenumber

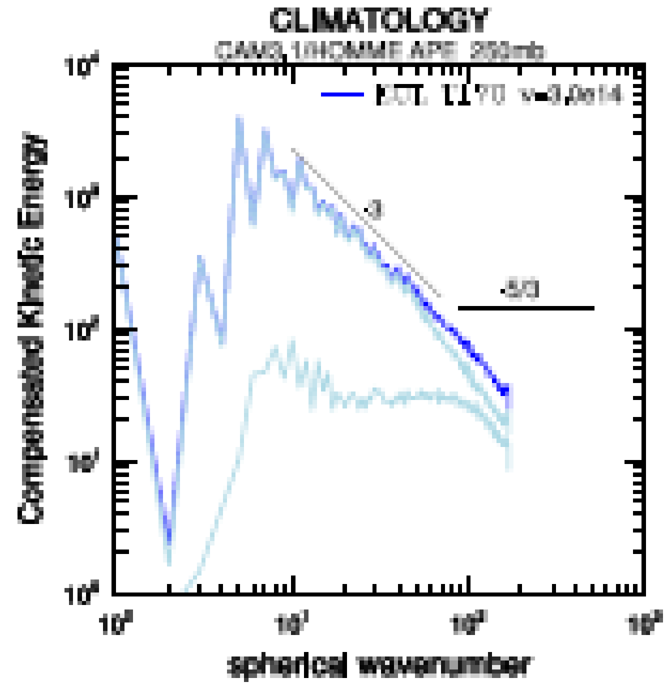
Rot
Div
KE

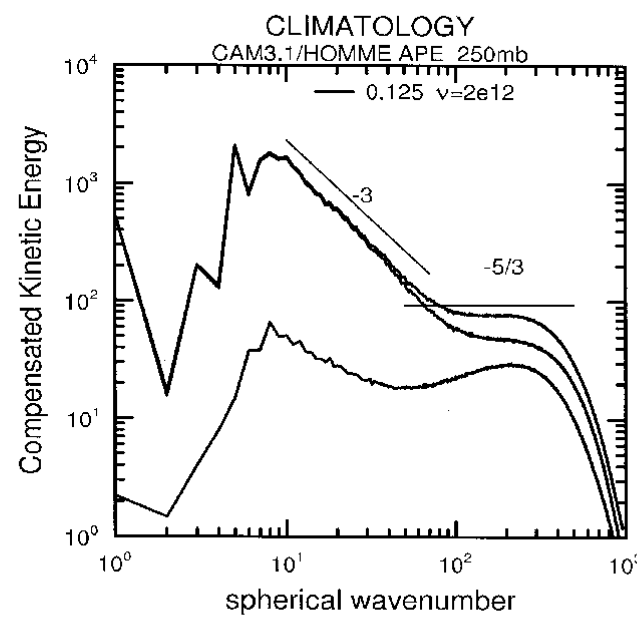


wavenumber

What (I believe) may be going on

- Divergent wind spectrum –5/3
- Due to balanced gravity waves
- Collision course breaks QG dynamics: vort & div same size.
- Not part of QG or balanced ordering
- Divergence amplified by moist processes
- Transition moves upscale
- Pathway to isotropy and 3D turbulence
- KE dissipation increases thru forward cascade





Some un-answered questions?

- What actually happens in the atmosphere ?
- Can we theorize without QG turbulence?
- Will predictability estimates change?
- How QGT dissipate?
- Are we parameterizing subgrid momentum exchange correctly? (Stochastic, bottleneck)
- How will modeled climate change with mesoscale variability?
- What is the role of moisture?

Rapid error growth in a turbulence closure inverse cascade

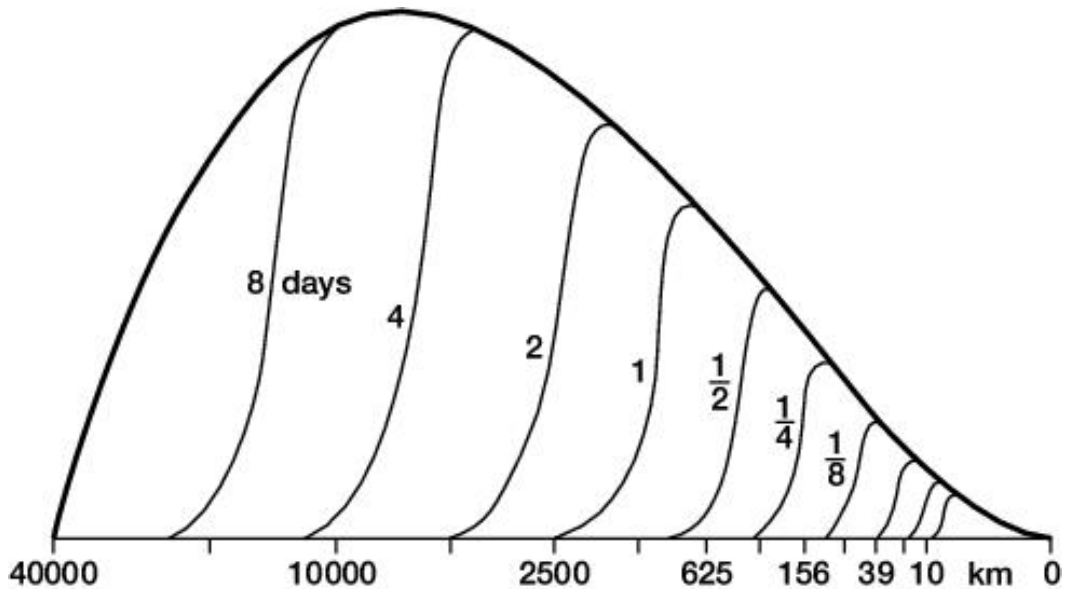


Figure 1: Growth of errors initially confined to smallest scales, according to a theoretical model (taken from a paper by E. Lorenz presented in AIP Conf. Proceedings #106). Horizontal scales on bottom; full atmospheric motion spectrum = upper curve.

Lorenz proposes
3 ways to estimate
Predictability

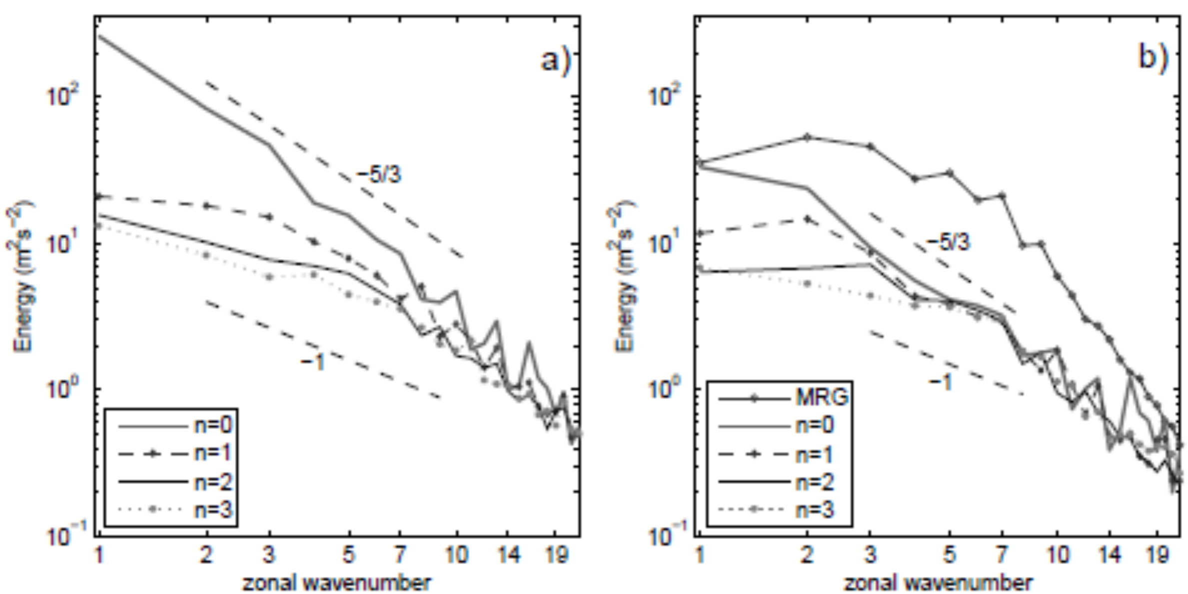
1) Model experiments
2) Analogues
3) Turbulence closure

All imperfect

Example of closure model

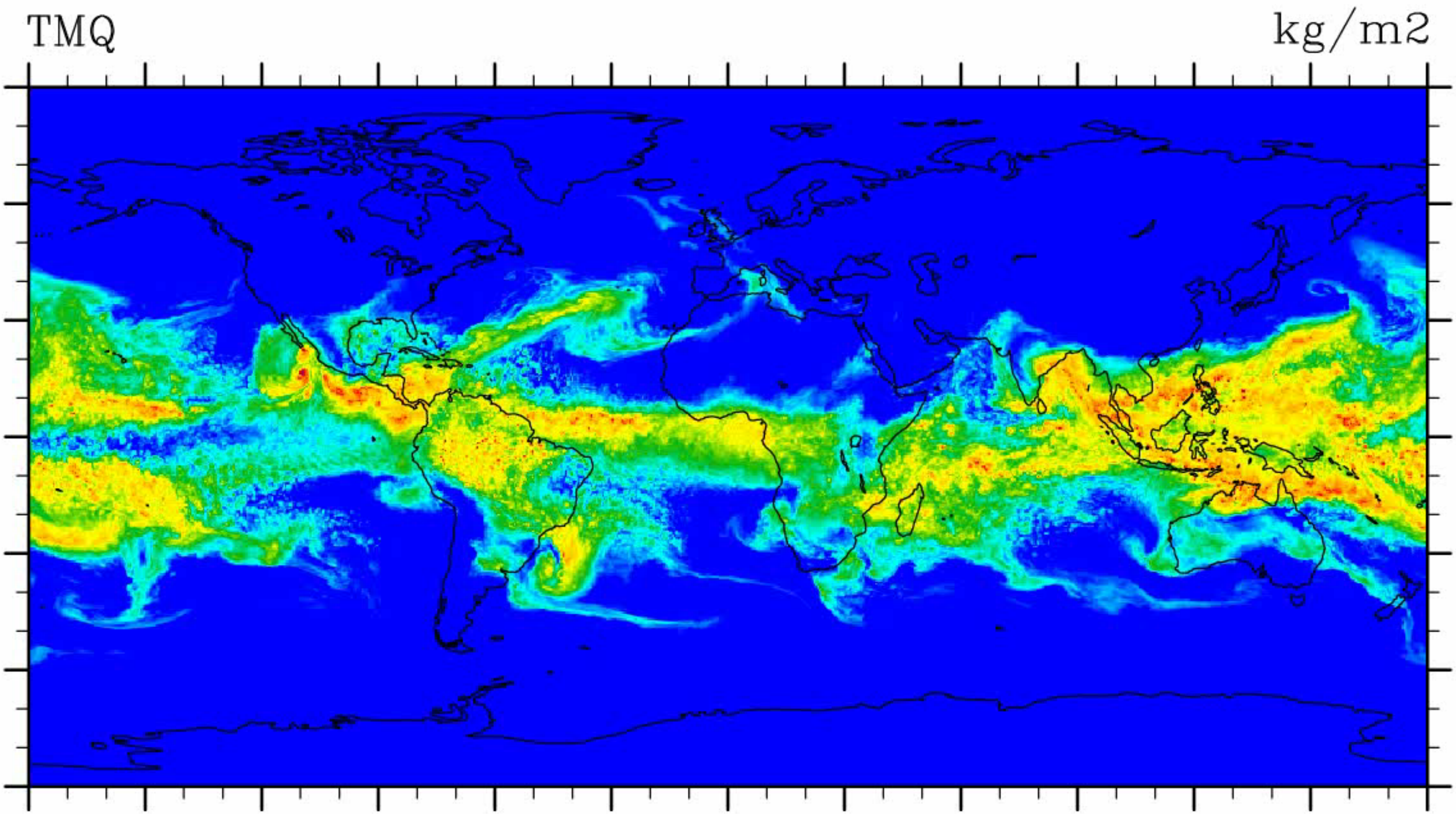
Moisture:Convectively Coupled Equatorial Gravity Wave spectra

Eastward Westward



Zagar et al (2009)

Climate Simulation in HOMME



The End

Thank you for your attention

Questions?

Recall:
QG equations with constant f

$$q_t + J(\psi, q) = 0,$$

$$q = \nabla^2\psi + (f^2 / S)\psi_{pp} \equiv L\psi$$

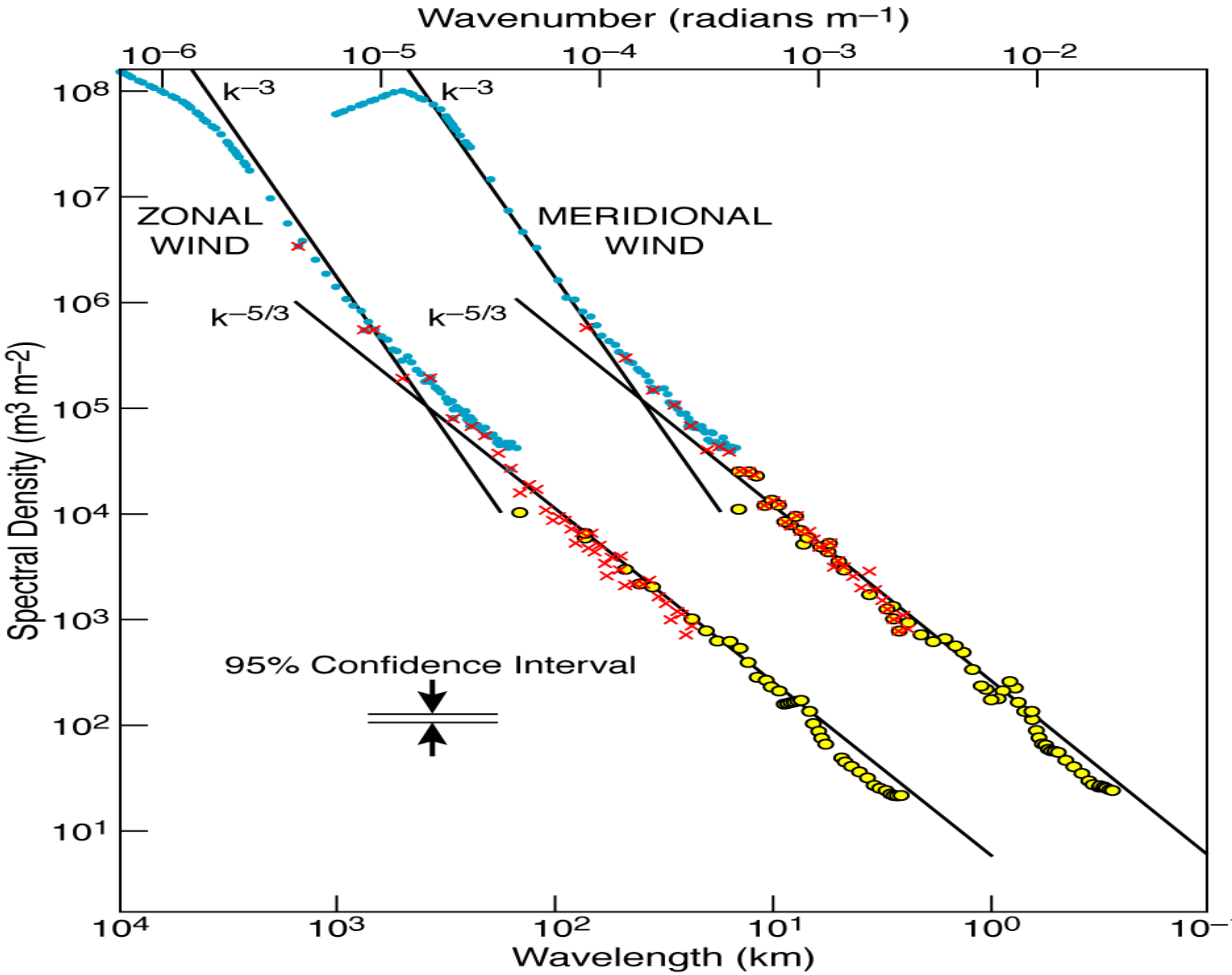
$$\psi_p = 0$$

$$p = 0, p_s$$

Note similarity to two–dimensional non-divergent governing equations. Isomorphic if variations in p is ignored and:

$$q \equiv \nabla^2\psi$$

Nastrom & Gage Spectrum
The real atmosphere not 2D or 3D



Atmospheric spectrum from analyzed data

Some time scales:

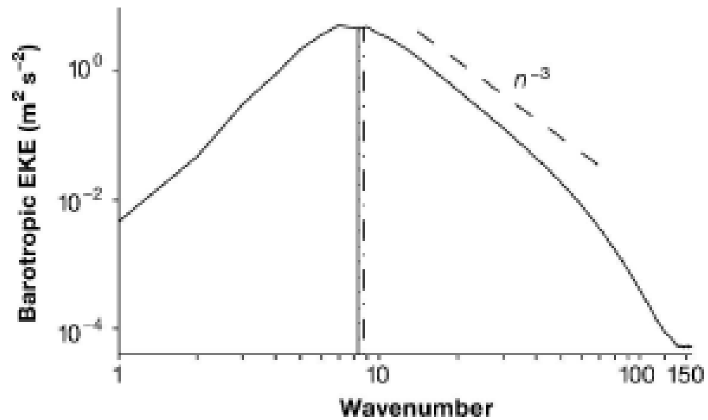
T_{eddy}~ $(E(k)k^3)^{1/2}$

T_{Rossby}~k/β

Rhines scale
Length at which:

$T_{edyy} = T_{Rossby}$

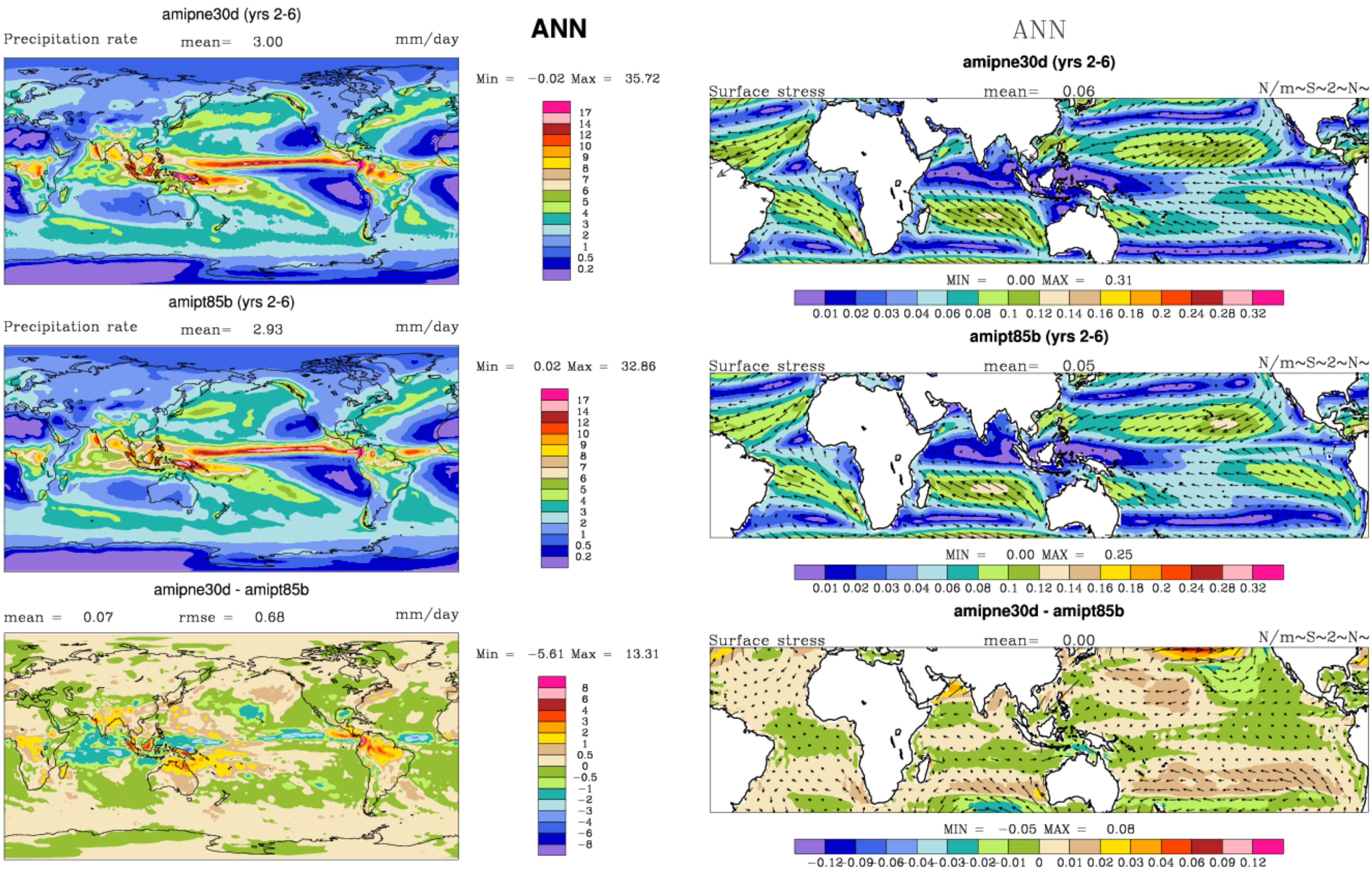
-5/3 range T_{eddy}~$k^{1/3}$
-3 range T_{eddy}~ const



Wavenumbers near 10 correspond to both the Rhines scale and the injection scale. Energy cascade to Large scales is inhibited by Rossby wave motion .

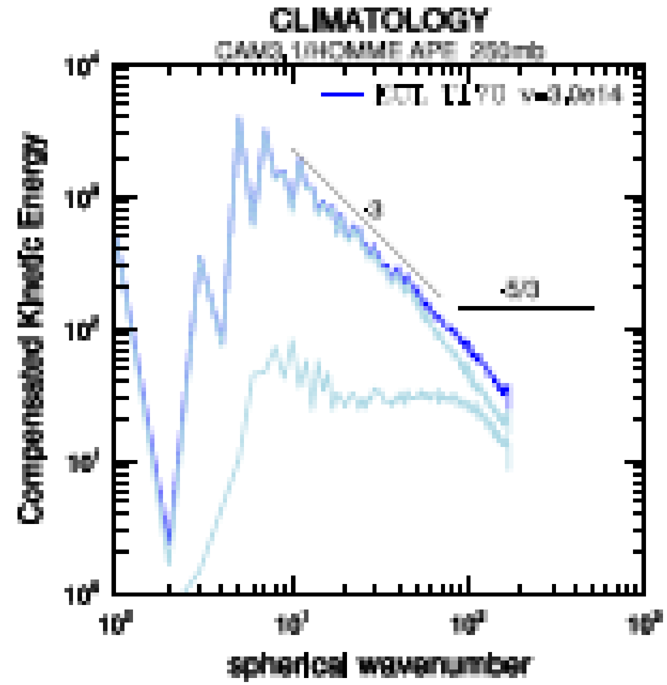
Few Rossby wave resonances
and few wavenumbers

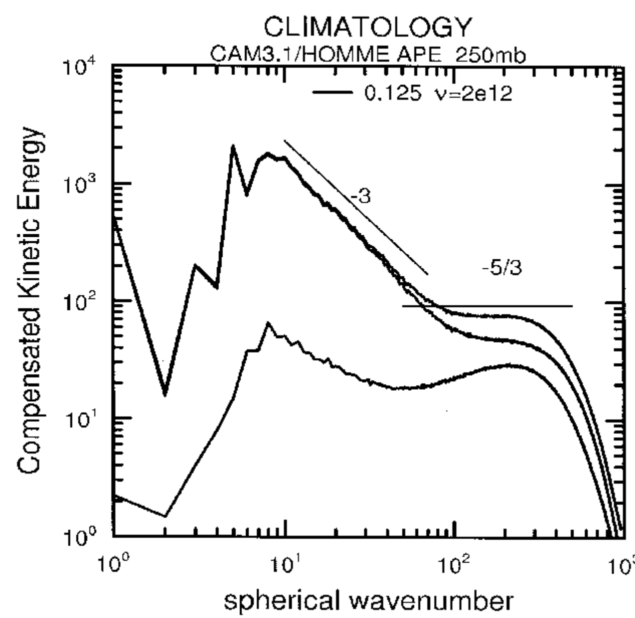
Climate Simulation in HOMME



What we think is going on

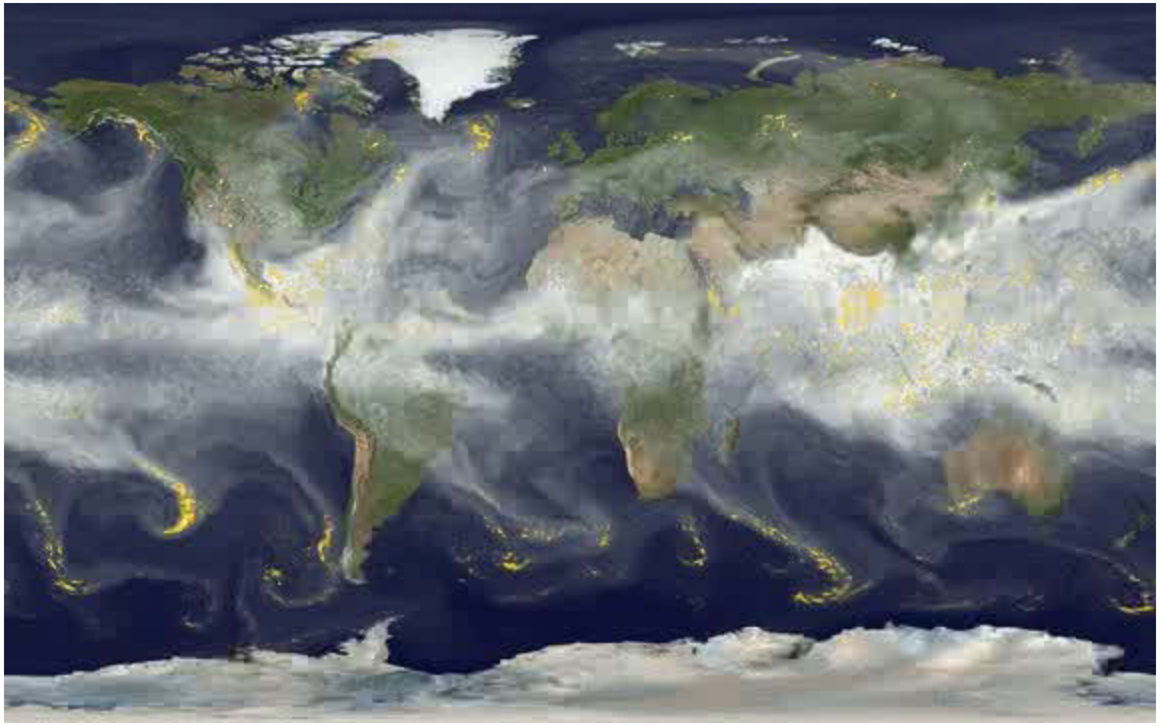
- Divergent wind spectrum –5/3
- Due to balanced gravity waves
- Collision course breaks QG dynamics: vort & div same size. Not part of QG ordering
- QG cannot be broken by small scale Rossby number
- Divergence amplified by moist processes
- Transition moves upscale
- Pathway to isotropy and 3D turbulence
- KE dissipation increases thru forward cascade





Predictability: How long can we accurately predict this?

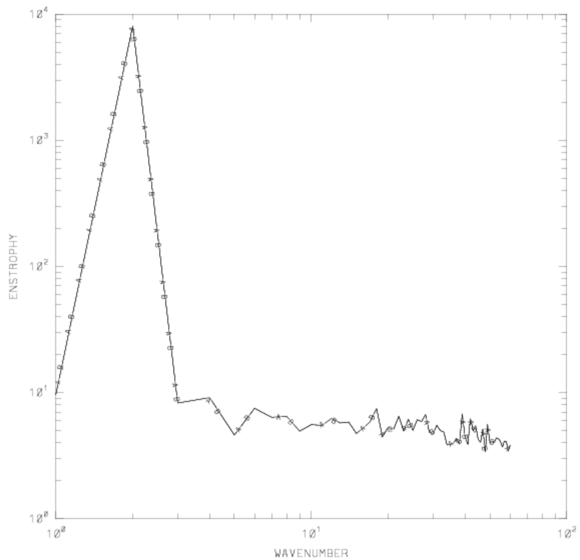
Water
Vapor
Channel



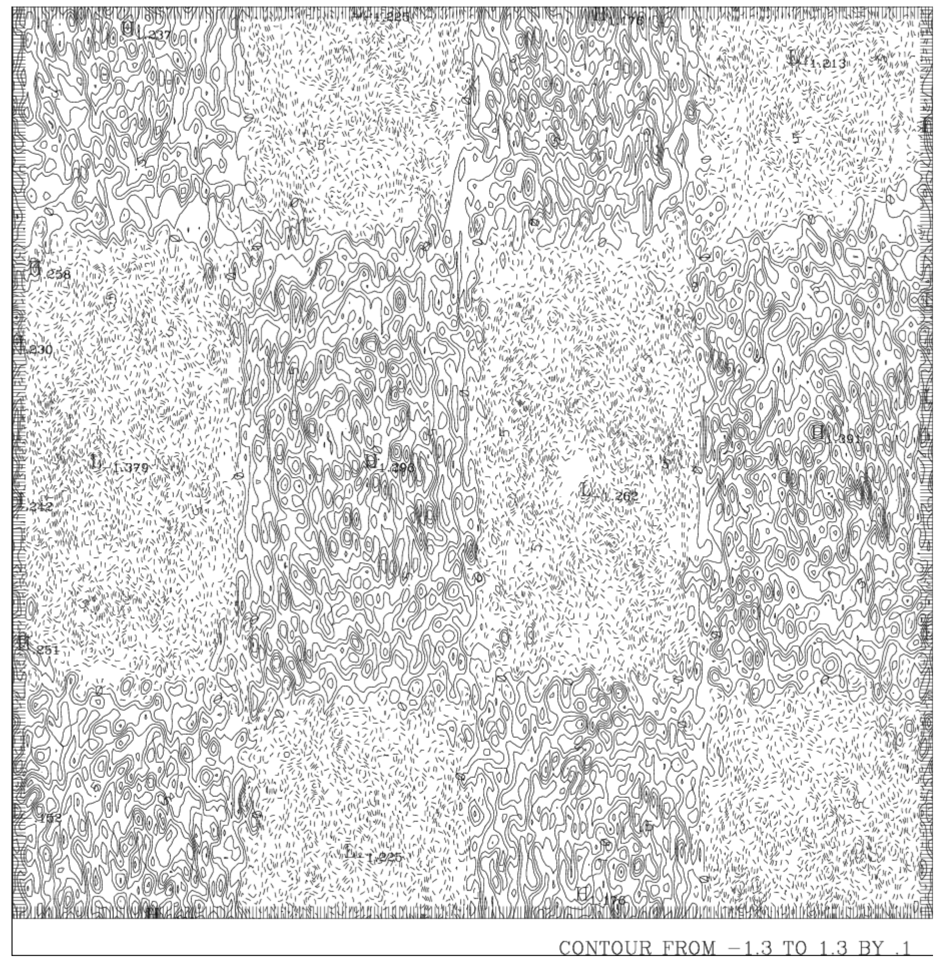
Chris Velden (U.Wisc/CIMSS)

'Prediction is hard-especially into the future'
Attributed to Neils Bohr

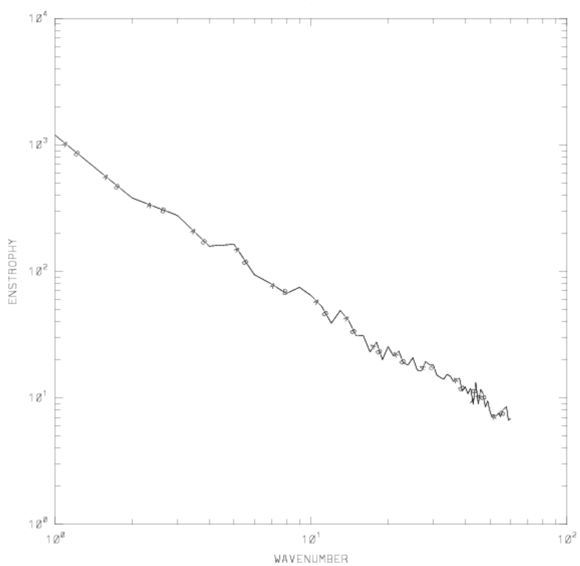
Evolving 2D Turbulence: simplest example



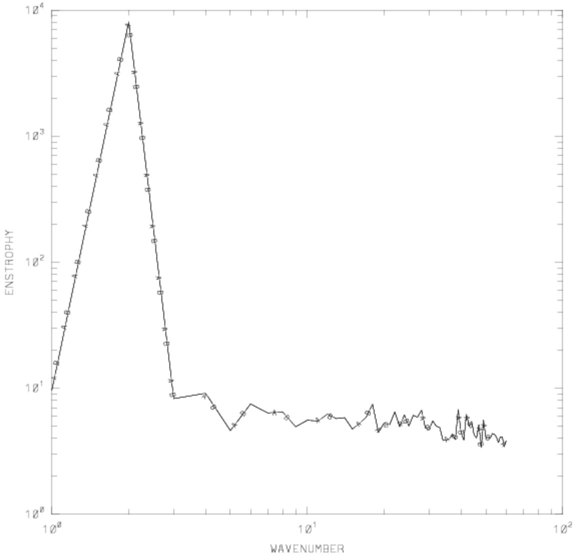
t=0



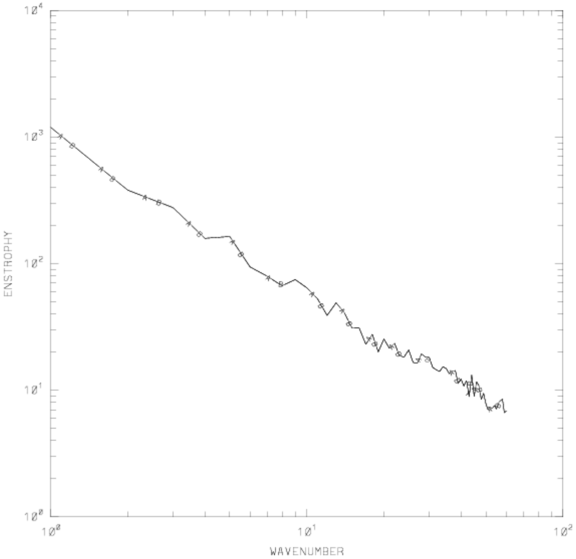
t=3-4



Evolving 2D Turbulence: simplest example



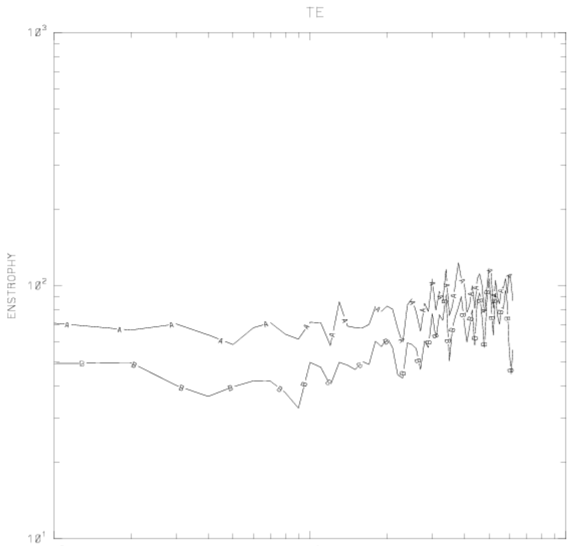
t=0



t=3-4

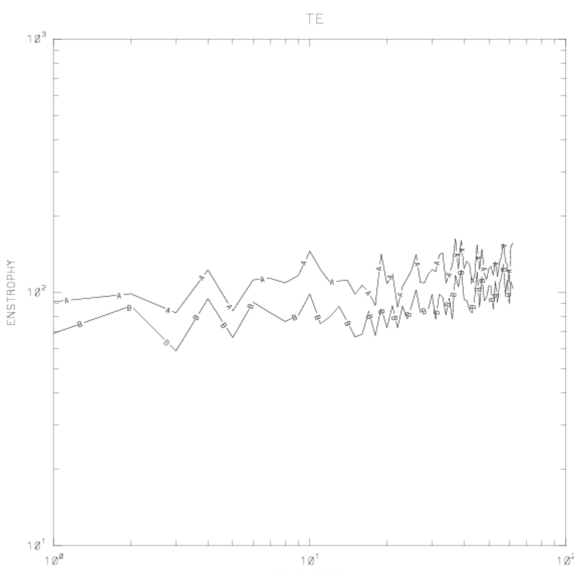
Backscattered vorticity tendency: fraternal twin

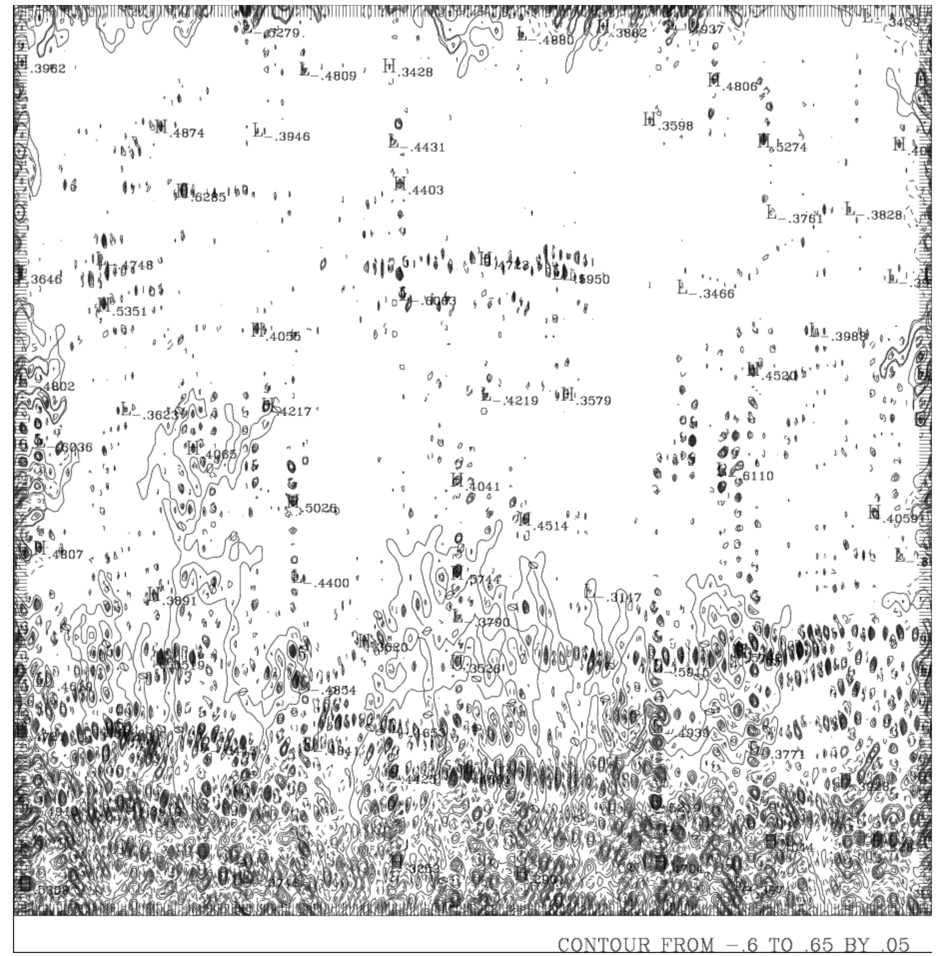
t=0



M(X,Y)
vs
M(X,0)

t=3-4





Important difference between 2D and 3D turbulence

2D turbulence

- Enstrophy cascade $[\eta]=T^{-3}$
- $[E(k)] = L^3T^{-2}$
- $E(k)= C_2\eta^{2/3}k^{-3}$
- $T^{-2} = [E(k)]/L^3 = k^3\ [E(k)]$
- $T(k)$~Constant
- Errors from small scale take longer and longer time to reach large scale (algebraic)

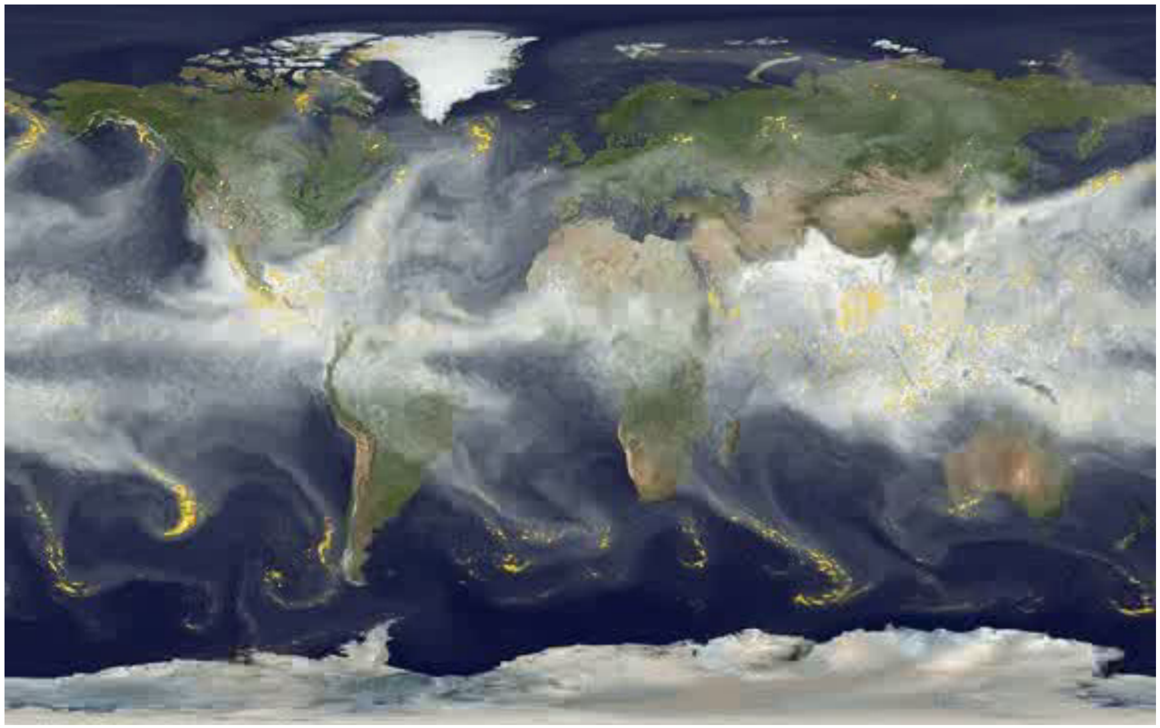
3D turbulence

- Energy cascade $[\varepsilon]=L^2T^{-3}$
- $[E(k)] = L^3T^{-2}$
- $E(k)= C_3\varepsilon^{2/3}k^{-5/3}$
- $T^{-2} = [E(k)]/L^3 = k^3\ [E(k)]$
- $T(k)$~const x $k^{-2/3}$
- Errors from small scale take a fixed time to reach large scale

Butterflies take a long time to influence large scale 2D weather
Too long compared to forcing at other scales

Can we describe this as two dimensional turbulence?

Water
Vapor
Channel

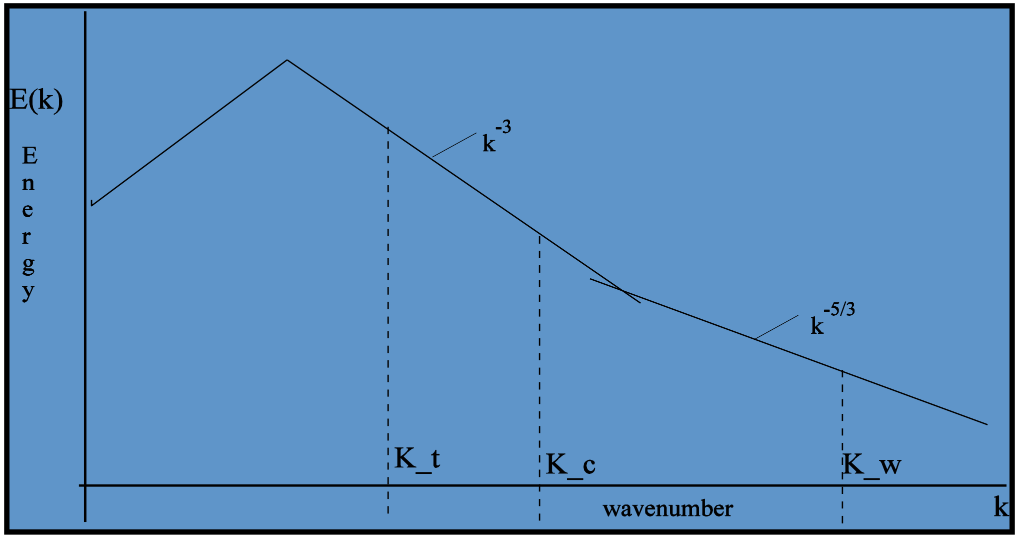


Horizontal turbulence in mid-latitudes
Atmosphere a thin fluid D/L<<1

Chris Velden (U.Wisc/CIMSS)

EXAMPLE: Spectral models and erroneous small scales

Schematic GFD energy spectrum with cutoffs



Simplest closure

$$\dot{Z}(k) = L_{k,j} Z(j) + N_{k,j,m} Z(j) Z(m)$$

$$X(k) \equiv Z(k); \quad k \le K$$

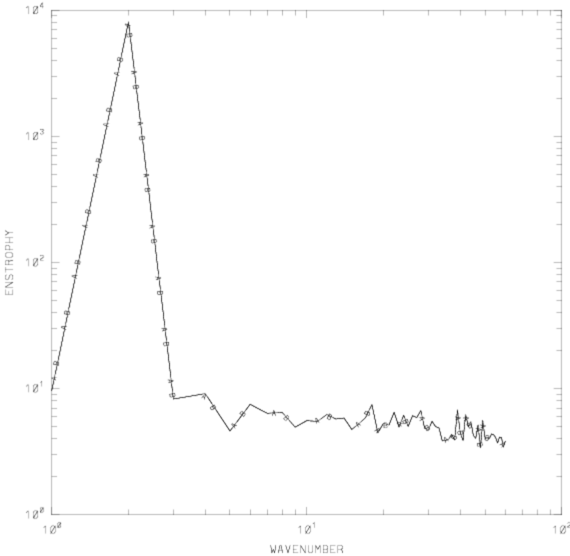
$$Y(k) \equiv Z(k); \quad k > K$$

$$\dot{\mathbf{X}} = \mathbf{M}(\mathbf{X}, \mathbf{Y})$$

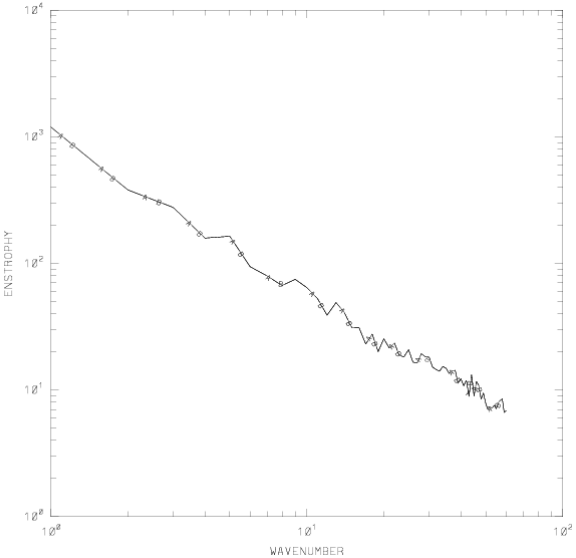
$$\dot{\mathbf{X}} \approx \mathbf{P}(\mathbf{X}) + (\mathbf{noise})\,?$$

Dynamics of Y(k)'s conditionally dependent on X(k)'s

Evolving 2D Turbulence: simplest example



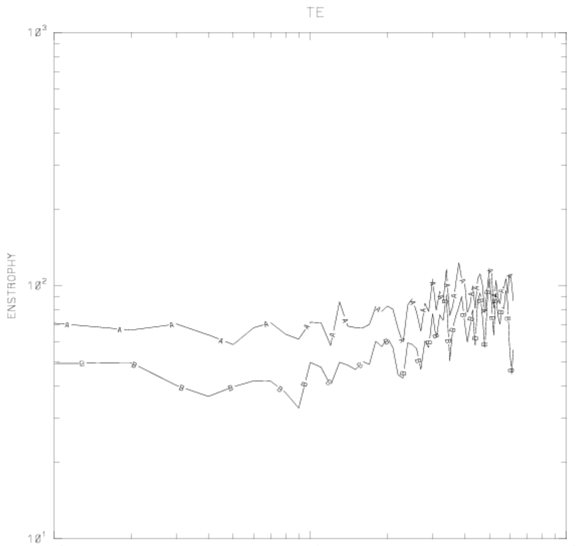
t=0



t=3-4

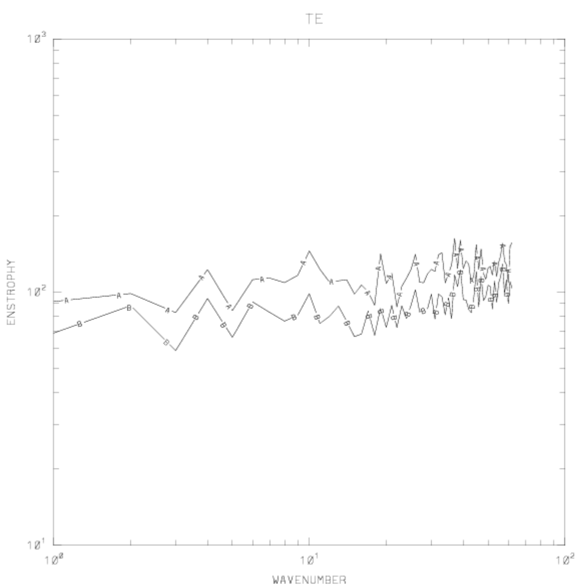
Backscattered vorticity tendency: fraternal twin

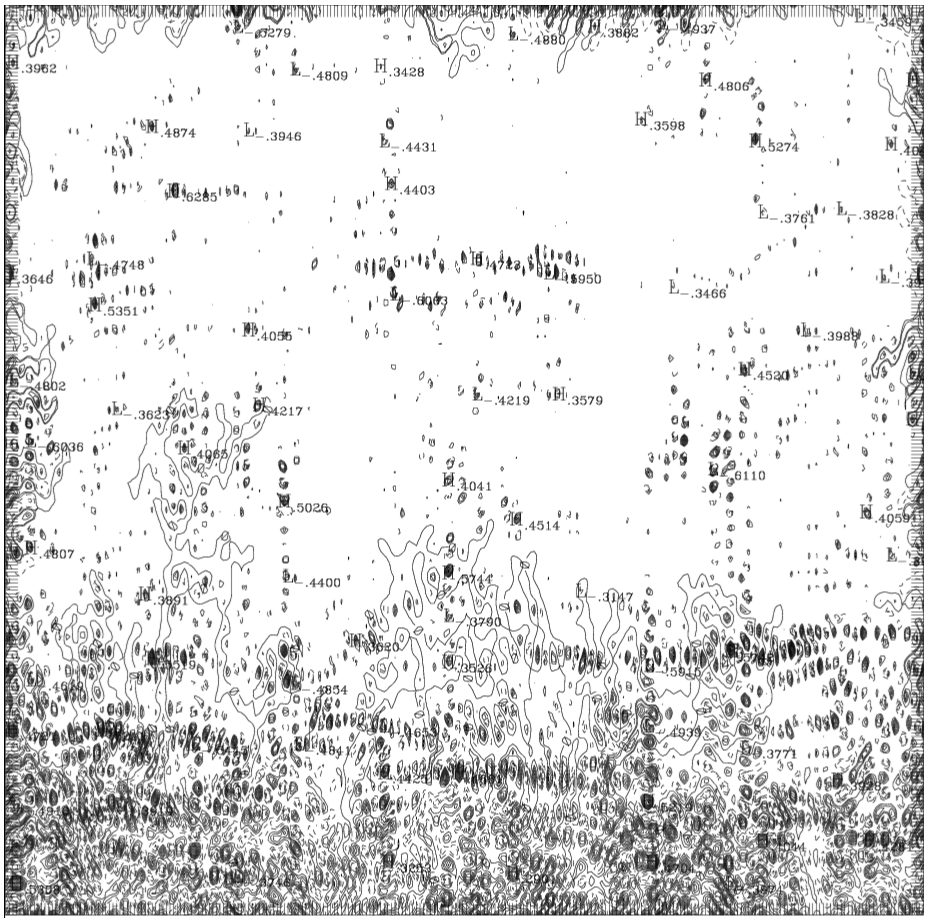
t=0



M(X,Y)
vs
M(X,0)

t=3-4





Probabilistic/Stochastic view of Predictability

Given a random dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},t) + \sqrt{D}\dot{\mathbf{w}}$$

Consider the evolution of

a density of states

$$p(\mathbf{x},t)$$

Fokker-Planck equation

$$\partial p/\partial t + \nabla \circ (\mathbf{f}p) = D\nabla^2 p$$

Singular vectors represent the time adjustment of the FP equation
Bred vectors effects of the initial state p(**x,**0)

How to get to quasi-geostrophic model

Start with Primitive Equations
In Non-dimensional Form

$$u_t - v + \phi_x = -R_o(uu_x + vu_y + \omega u_p) + \frac{\beta L}{f_0} yv,$$
$$v_t + u + \phi_y = -R_o(uv_x + vv_y + \omega v_p) - \frac{\beta L}{f_0} yu,$$
$$\phi_{pt} + B\omega = -R_o(u\phi_x + v\phi_y + \omega\phi_{pp} + \frac{\omega}{p}(1-\kappa)\phi_p),$$
$$u_x + v_y + \omega_p = 0,$$

R=U/fL the Rossby Number
B=D(N/fL)**2 the Burger Number

For small Rossby Number Asymptotic Expansion and Resonance Condition Gives:

$$\begin{aligned}
&-\Pi_y(u^0_T + u^1_{t'} - v^1 + \phi^1_x - NL^0_u - \widehat{\beta} y v^0) + \\
&\Pi_x(v^0_T + v^1_{t'} + u^1 + \phi^1_y - NL^0_v + \widehat{\beta} y u^0) + \\
&\Pi_p B^{-1}(\phi^0_{pT} + \phi^1_{pt'} + B\omega^1 - NL^0_\phi),
\end{aligned}$$

which vanishes for any arbitrary scalar function $\Pi(x,y,p)$. The standard trick of integration by parts is performed in order to move the spatial derivatives on Π to the field variables (u, v, ϕ). Recalling that the resonant terms are those involving ONLY the rotational modes, the vanishing of the integral requires:

$$\frac{\partial}{\partial t'} q^1 + \frac{\partial}{\partial T} q^0 = -\overrightarrow{V}_g \cdot \nabla q^0 - \widehat{\beta} v_g,$$

where $\overrightarrow{V}_g \equiv (-\phi^0_y, \phi^0_x)$ and $q^0 \equiv \phi^0_{xx} + \phi^0_{yy} + (\frac{\phi^0_p}{B})_p$. To avoid linear growth in the fast time, t', equate the 2nd term on the left with the whole RHS and

$$q_t + J(\psi, q) = 0$$

Same form as 2D vorticity equation. Invert 3D elliptic operator

Important difference between 2D and 3D turbulence

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- Enstrophy cascade $[\eta]=T^{-3}$
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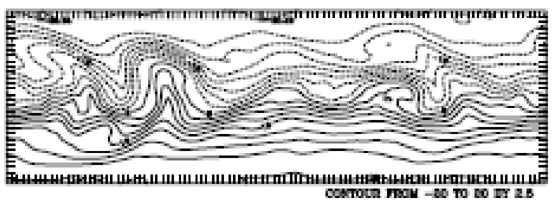
Butterflies take a long time to influence large scale 2D weather
Too long compared to forcing at other scales

Singular vector analysis of fraternal twin error growth

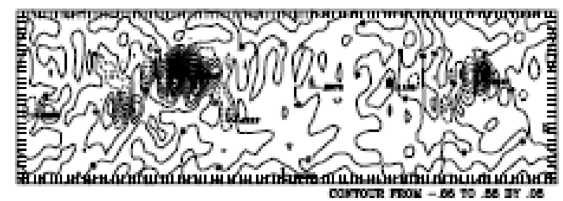
Singular vectors

vorticity errors

QG model State

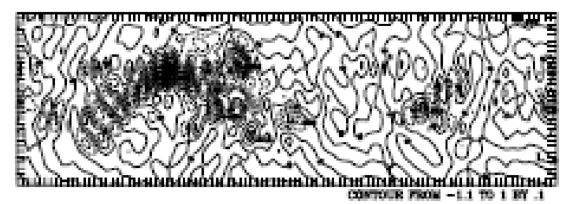


Error growth



Error growth

Leading SV



Rapid growing structures organize the stochastic backscatter

Rationale

Why examine flatland (2D/QG) turbulence ?

DYNAMICAL MODEL OF ATMOSPHERE

- Theory : cascades,energy-enstrophy, scale interactions
- Practice: Atmospheric spectrum matches theory
- Test: Predictability: Can small scales contaminate weather prediction?

"In theory, there is no difference between theory and practice.
But, in practice, there is." – Jan L.A. van de Snepscheut